12835.a.86. ~~8503.df.21~~

ARITHMETIC

Made familiar and eaſy to
Young *Gentlemen* and *Ladies*.

Being the

SECOND VOLUME

OF THE

Circle of the Sciences, *&c.*

Publiſhed by the KING'S *Authority*.

The FIFTH EDITION.

LONDON:
Printed for THOMAS CARNAN, Succeſſor to
Mr J. NEWBERY, in St. Paul's Church-yard.
MDCCLXXXVIII.

[Price One Shilling Bouud.]

BRITISH MUSEUM
14 JUL 51

To His Royal Highneſs

Prince EDWARD AUGUSTUS,

THIS

ART of ARITHMETIC

Is humbly inſcrib'd

BY

His Royal Highneſs's

moſt obedient Servant,

JOHN NEWBERY.

11

BRITISH MUSEUM
14 JUL 51

PREFACE.

I Foreſee but one Objection that may be made againſt the following Compendium *of the moſt uſeful Rules in Arithmetic*, viz. *that no Notice is therein taken of the Doctrine of* Vulgar Fractions: *The Reaſon of which Omiſſion is, that I thought them too difficult for the tender Capacities of thoſe, for whoſe Service this little Work is principally intended; and beſides, moſt common Buſineſs may be carried on with the Knowledge of* Whole Numbers *only, or ſuch a ſmall Skill in* Fractions *as may be ſoon acquired whenever it is found neceſſary. In general, I hope the Rules and Examples I have given are laid down and ex-*

plained in ſuch an eaſy and familiar Manner, as to fall within the Comprehenſions of Youth, and render the Work a proper Introduction to the Study of the more abſtruſe Parts of Arithmetic. But inſtead of ſpending Time in ſetting forth the Merit of the Performance, the uſual Buſineſs of Prefaces) *I ſubmit it entirely to the Reader's Judgment, and ſhall employ a few Pages in giving him a ſhort Account of the Riſe and Progreſs of the* Art of Numbers.

We have very little Intelligence indeed about the Origin or Invention of Arithmetic, *Hiſtory neither fixing its Author nor the Time. In all Probability, however, it muſt have taken its Riſe from the Introduction of Commerce, and conſequently be of* Tyrian *Original.*

From Aſia *it paſſed into* Egypt, (*by Means of* Abraham *according to* Joſephus) *where it was greatly cultivated and improved, inſomuch that a large Part*

Part of their Philoſophy and Theology ſeems to have turned altogether upon Numbers: And Pythagoras, *who learnt this Science from the* Egyptians, *attempted to explain all Things in Nature by Numbers, affirming that the Knowledge of Numbers was the Knowledge of the Deity.*

From Egypt *Arithmetic was tranſmitted to the* Greeks, *who handed it forwards, with great Improvements which it had received from their Aſtronomers, to the* Romans; *and from them it ſpread over* Europe *with their Conqueſts.*

The Arabians, *by the happy Invention of the Cypher, made the Way to the Science of Numbers much more eaſy than it was before; and to them we are indebted for the Art of* Algebra, *which, after having been long neglected and little underſtood, has of late been revived and very much improved in* Europe.

In truth the Arithmetic *of the Ancients fell far ſhort of that of the Moderns, who in the laſt and the preſent Century have applied themſelves to the Study of this excellent Science with ſo much Succeſs, that they ſeem to have carried it to its utmoſt Perfection.*

As to the Method of Computation in Uſe amongst us by a Series of ten Characters, ſo that the Progreſſion is from Ten to Ten, it was utterly unknown to the Greeks *and* Romans, *and is ſaid to have been firſt introduced into* Europe *about the End of the tenth Century, by* Gerbert, *afterwards Pope under the Name of* Sylveſter II. *who borrowed it from the* Moors *of* Spain. *No doubt it took its Origin from the Way of reckoning on the Fingers, which was uſed before* Arithmetic *was brought into an Art: And hence, perhaps, the ſingle Figures,* 1, 2, 3, &c. *have obtained*

tained the Name of Digits, *from the* Latin *Word* Digitus, *a Finger.*

The Eaſtern Miſſionaries aſſure us, that to this Day the Indians *are very expert at computing on their Fingers, without the Uſe of Pen and Ink. The* Chineſe *do not much regard Rules in their Calculations, inſtead of which (Father* le Comte *tells us) they uſe an Inſtrument made of a Plate a Foot and a half long, acroſs which are fitted ten or twelve Iron Wires, whereon are ſtrung little round Balls: By drawing theſe together, and diſperſing them again one after another, they calculate with great Eaſe and Expedition; and have alſo the Manner of proving the Truth of the Operation. Add to this, that the Natives of* Peru, *who make their Computations by the different Arrangement of Grains of Maize, are ſaid to equal any* European, *with all his Rules, both in Sureneſs and Diſpatch.*

I mention theſe Things only as Matters of Curioſity, and not with any Deſign to depreciate the noble Science of Arithmetic, *the Study of which I heartily recommend to the Youth of both Sexes, as an Attainment that will be of excellent Service to them almoſt in every Station of Life.*

BRITISH MUSEUM 14 JUL 61

ARITHMETIC.

Its Definition, Uſefulneſs, and principal Rules.

Q. WHAT is Arithmetic?

A. The *Art of Numbering*, whereby we reckon or calculate truly, and with Eaſe and Expedition.

Q. Of what Uſe is it?

A. It is neceſſary for the carrying on of Trade and Buſineſs, for the Management of an Eſtate, and many other Affairs of Life. Without the Knowledge of *Arithmetic* we are liable to be impoſed upon in our Dealings with Mankind, or defrauded by careleſs Servants and unjuſt Stewards: But by a competent Skill in Numbers theſe Inconveniencies may be avoided, and we may probably increaſe our Store, and become rich and honourable. Even the Ladies themſelves, who have generally the Care of the domeſtic Expences of a Family, ought therefore to have a proper Share of this uſeful Accompliſhment.

Q. Which are the chief Rules in Arithmetic?

A. *Numeration*, *Addition*, *Subtraction*, *Multiplication*, and *Diviſion*.

CHAP. I.

Of NUMERATION, *or* NOTATION.

Q. WHAT is NUMERATION?

A. By *Numeration*, or *Notation*, as it is called by ſome late Arithmeticians, we are taught how to read or write down any Number propoſed.

Q. How is that to be done ?

A. By the ten following Figures or Characters, viz.

One, Two, Three, Four, Five,
1 2 3 4 5
Six, Seven, Eight, Nine, Cypher.
6 7 8 9 0

Q. But how can theſe few Figures expreſs all Numbers whatſoever ?

A. By having different Values or Significations ſet upon them according to the Place they ſtand in ; for tho' each of them when alone has a fix'd and certain Value, as 1 *one*, 2 *two*, &c. yet the ſame Figure, when placed along with others, may expreſs very different Numbers ; as 1 may ſignify *one* only, or *ten*, or a *hundred*, *a thouſand*, ten *thouſand*, &c.

Q. How then ſhall I know the Value of any Figure ?

A. By obſerving the Place it ſtands in, and remembering, that if it be alone, or in the firſt Place on the Right Hand, it ſignifies ſo many *Units* or *Ones* as the Figure ſimply expreſſes; as, 5 denotes five *Units:* But place another Figure after it, and it ſignifies five *Tens*, or *Fifty*; as 51 is *Fifty-one*. So if another Figure were added after it, the 5 would ſignify five *Hundreds*, that is, juſt ten Times as many as it did before; for each Remove from the Right Hand towards the Left increaſes the Value of a Figure in a tenfold Proportion, from Ones to Tens, from Tens to Hundreds, from Hundreds to Thouſands, from Thouſands to Tens of Thouſands, &c.

Q. What is the Uſe of the round Character you call a *Cypher?*

A. Though of itſelf it ſignifies *Nothing*, yet being placed after other Figures it gives Value to them in the above-mentioned Proportion: Thus 9, when alone, ſtands for *Nine* only; but being removed one Step towards the Left Hand, by adding a Cypher after it, it ſignifies ten times as many, that is, 90 Ninety. But what has been ſaid will appear more plainly by a due Attention to the following Table.

Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thouſands.	Tens of Thouſands.	Thouſands.	Hundreds.	Tens.	Units.
								1
							1	2
						1	2	3
					1	2	3	4
				1	2	3	4	5
			1	2	3	4	5	6
		1	2	3	4	5	6	7
	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	9

From this Table you may obſerve, that 1 ſtanding alone ſignifies a ſimple Unit, or *One* only; but in the next Row of Numbers, being placed on the Left of the Figure 2, it ſtands for one *Ten*, or ten Units, and both of them put together make up *Twelve*. In the next Row you find the Figure 1 ſignifies *one Hundred*, being removed into the third Place from the Right Hand, with the Figure 2 in the Place of Tens, and 3 in that of Units: the Sum of theſe three Figures therefore, as they ſtand in a Row, is 1 Hundred, 2 Tens or Twenty, and 3 Units or Ones; that is, *One hundred and twenty-three*. Again: In the next Row of Figures, the 1 is the fourth Place from the Right Hand, which makes it ſtand for *One Thouſand*, having 2 after it in the Place of Hundreds, 3 in the Place of Tens, and 4 in that of Units; which expreſs'd together, as they ſtand in a Row, 1234, amounts to *One thouſand two hundred and thirty-four.* Thus exerciſing yourſelf at firſt in the ſmaller Numbers, you may proceed to the greater, till you are perfect in the whole Table, obſerving how the Value of Figures increaſes as they are removed from the Right Hand towards the Left, and remembering the Names of the Places: And by this Means you will ſoon be able to expreſs in Words, or to write down in Characters, any Number you will have occaſion for in the common Buſineſs of Life.

Q. But how ſhall I know how to write in Figures any Number that is not in the Table, ſuppoſe it to be *Two hundred and Twenty-five?*

A. As you begin with Hundreds, which you ſee by the Table muſt ſtand in the *third* Place towards the Left Hand, it is plain that three Figures will be neceſſary to expreſs the Number propoſed. Therefore firſt ſet down 2 for the Hundreds; after that 2 for the Tens, becauſe there are two Tens in twenty; then 5 in the Place of Units, and the Figures will expreſs the Number you deſire, 225.

Q. If Cyphers be mixed amongſt Figures, how muſt they be reckoned?

A. Remember that of themſelves they ſignify *nothing*, but increaſe the Value of other Figures that ſtand before them: Suppoſe therefore you meet with this Number 400, as there is a Cypher in the Place of Units, and another in the Place of Tens, theſe are of no Signification; but the 4 denotes *four hundred*, being advanced to the Place of Hundreds by the two Cyphers. So if you would expreſs *two hundred and five* in Figures, you muſt ſet down 2 for the Hundreds, and having no Tens make a Cypher in the Place of Tens, and a 5 in that of Units

nits, and you have the Number propoſed, 205.

Q. How is the loweſt Row of Figures in the Table (*viz.* 123456789) to be expreſſed in Words at length?

A. *One hundred and twenty-three Millions, four hundred fifty-ſix Thouſands, ſeven hundred eighty-nine.* N. B. In theſe long Numbers it is uſual to make a Comma or Stop between every three Figures, from the Right Hand to the Left, whereby the Millions, Thouſands, &c. are more eaſily diſtinguiſhed; as,

231,564,970
24,301,025
5,210,344

Which the young Scholar may endeavour to expreſs in Words for his Improvement, after he is well acquainted with ſmaller Numbers.

CHAP. II.

Of ADDITION.

Q. WHAT is ADDITION ?

A. It is the *putting together of ſeveral Numbers, ſo as to find their total Amount*, that is, one Number equal to them all.

Q. What is firſt to be obſerved in this Operation ?

A. In ſetting down the Numbers to be added, Care muſt be taken to place every Figure in its proper Column, that is, Units under Units, Tens under Tens, Hundreds under Hundreds, &c. as in the following Examples :

Books.	*Nuts.*	*Pins.*
1	24	172
7	80	345
8	61	201
3	43	129
In all 19	In all 208	In all 847

Q. In what Manner do you add theſe Numbers together?

A. The firſt Example, or that on the Left Hand, is a Row of Single Figures, which I ſum up in this Manner: Beginning at the Bottom, and going upwards, I ſay, 3 and 8 is 11, and 7 is 18, and 1 is 19; and having no more Figures to add, I ſet down 19 at the Bottom as the total Amount. —In the ſecond Example, I likewiſe begin at the Bottom of the Rank of Units (which is always to be done) and ſay, 3 and 1 is 4, and a Cypher is *Nothing*, and 4 is 8, which being leſs than 10, I ſet down under the Units, and proceed to the Left-hand Column, ſaying, 4 and 6 is 10, and 8 is 18, and 2 is 20; which I put down underneath on the Left of the 8, and find the Amount of the ſeveral Parts or Figures to be 208.—In like Manner I begin with the Units of the third Example, ſaying, 9 and 1 is 10, and 5 is 15, and 2 is 17, which being 7 more than ten, I ſet down 7 under the Row of Units, and carry 1 for the 10 to the next Column on the Left, ſaying, 1 and 2 is 3, the Cypher is Nothing, and 4 is 7, and 7 is 14. Here again, the Number 14 being 4 more than 10, I put down 4 under the Row of Tens, and carry 1 to the Column of Hundreds, ſaying, 1 and 1 is 2, and 2 is 4, and 3 is 7, and 1 is 8, which 8 I place underneath, and find the Total to be 847.

Q. Is *One* for *Ten* to be always carried from any Row of Figures to the next on the Left Hand, as in the laſt Example?

A. In the Addition of Numbers of one Denomination, that is, of one Name or Kind, whether they be all *Gallons*, *Pounds*, *Miles*, *Acres*, or whatever, (which is called *Simple* Addition) you muſt always carry 1 for every 10 that you find in the Row of Units to the next Row of Tens, and the like from the Tens to the Hundreds, from the Hundreds to the Thouſands, *&c.* till you have gone through all the Rows, be they ever ſo many. So if the Amount of the Column of Units be juſt 10 and no more, ſet down a Cypher underneath the Units, and carry 1 to the Tens; but if they amount to 20, a Cypher muſt be put down and 2 carried, becauſe there are two Tens in Twenty. In like

Manner, if the Amount of a Right-hand Column be 45, the 5 muſt be placed underneath in the Total, and 4 carried to the Column on the Left, becauſe there are four Tens in Forty. When you come to the laſt Column, the Amount of it muſt be ſet down, be it what it will, becauſe the Operation is compleated, and the Tens can be carried no farther. Take an Example or two more for Practice.

	Boys.	*Horſes.*
	2907	4572
	315	8601
	8549	209
	2660	74
Total	14431	13456

Let us now proceed to *Compound* Addition, or that of Numbers of different Denominations, as we find in *Money*, *Weight*, *Meaſure*, &c. And firſt of *Money*.

Q. What is to be obſerved in caſting up Sums of Money?

A. Remember that in *England* we keep our Accounts in *Pounds*, *Shillings*, *Pence*, and *Farthings*; and therefore take particular Notice, that

4 Farthings	make	1 Penny.
12 Pence		1 Shilling.
20 Shillings		1 Pound.

Beſides, for your greater Eaſe in this Sort of Addition, it will be proper to get by Heart the following:

PENCE-TABLE.

20*d.*	is	1*s.*	8*d.*
30	—	2	6
40	—	3	4
50	—	4	2
60	—	5	0
70	—	5	10
80	—	6	8
90	—	7	6
100	—	8	4
110	—	9	2
120	—	10	0

Q. What is farther to be obſerved?

A. In ſetting down the Numbers to be added together, be ſure to place Pounds under Pounds, Shillings under Shillings, Pence under Pence, and Farthings under Farthings; and you may put an *l.* over the Column of Pounds, an *s.* over the Shillings, a *d.* over the Pence, and a *q.* or *qr.* over the Farthings.

Q. What is the meaning of thoſe Letters?

A. They are the firſt Letters of the *Latin* Words *Libra* a Pound, *Solidus* a Shilling, *Denarius* a Penny, and *Quadrans* a Farthing.

Q. Let me ſee an Example according to theſe Directions.

A. I owe to one Perſon *Three Pounds ſix Shillings and Six-pence Farthing*, to another, *One Pound four Shillings and Ten-pence Half-penny*, and to a third *Two Pounds Eleven Shillings and three*

three Farthings. In order to know what theſe ſeveral Debts amount to, I ſet them down in the following Manner, and find the Total to be Seven Pounds two Shillings and Five-pence Half-penny.

l.	*s.*	*d.*	*q.*	
3	6	6	1	
1	4	10	2	
2	11	0	3	
7	2	5	2	Total.

Q. But how do you proceed in the Operation?

A. I begin with the Farthings, and ſay, 3 and 2 is 5, and 1 is 6; then conſidering that 4 Farthings make 1 Penny, I ſet down only 2 Farthings under that Column, and carry 1 for the 4 to the Column of Pence. Then (paſſing over the Cypher) I ſay, 1 that I carry and 10 is 11, and ſix is 17;

which being 5 Pence more than a Shilling, I ſet down 5 under the Pence and carry 1 for 12 to the Column of Shillings; ſaying, 1 that I carry and 11 is 12, and 4 is 16, and 6 is 22. Here again I conſider that 22 Shillings is 2 more than a Pound; therefore I ſet down 2 under the Shillings, and for the 20, I carry 1 to the Column of Pounds; ſaying, 1 I carry and 2 is 3, and 1 is 4, and 3 is 7, which I ſet down under the Pounds, and ſo the Work is compleated.

Q. Is the ſame Method to be obſerved in all Sums of this Nature?

A. Yes; always remember for every four Farthings to carry 1 Penny, for every 12 Pence to carry 1 Shilling, and for every 20 Shillings to carry 1 Pound; ſetting down, as you go along, the overplus Farthings, Pence, and Shillings (if ſuch their be) under their proper Columns.

Q. Are Farthings always expreſs'd as in the Example, with a *q* over them?

A. No; they are commonly ſet cloſer to the Column of Pence, and are expreſs'd as follows, *viz.* $\frac{1}{4}$ is one Farthing, $\frac{1}{2}$ is two Farthings or an Halfpenny, $\frac{3}{4}$ is three Farthings. Theſe are call'd *Fractions*, and are to be caſt up in the ſame Manner as before; as you will find in theſe Examples:

		s.		d.
Laid out for Apples		1	—	6$\frac{1}{2}$
Gingerbread		2	—	4$\frac{3}{4}$
— Marbles	—	0	—	9
Oranges	—	1	—	5$\frac{1}{4}$
Total		6	—	1$\frac{1}{2}$

Here I caſt up the Farthings, as in the former Example; and finding them to be 6, or 2 more than a Penny, I ſet down 2 Farthings or a Halfpenny thus $\frac{1}{2}$, and carry one to the Pence, which being added together, amount to 25. Then conſidering that in 25 Pence there are twice 12, (that is, two Shillings) and 1 Penny over, I put down the odd Penny under the Column of Pence, and carry 2 to the Shillings; which amounting to 6, I place 6 under that Column, and find the Total to be as above expreſs'd.— Take one Example more for your own Practice.

	l.	*s.*	*d.*
Laid out	22 —	16 —	8$\frac{1}{4}$
and	33 —	10 —	4
and	29 —	3 —	7$\frac{3}{4}$
Total	85 —	10 —	8

N. B. When the Column of Pounds has two or more Figures in a Row, add them up as Numbers of one Denomination; that is, for every

10 in the Place of Units carry 1 to that of Tens, and the like from the Tens to the Hundreds, *&c.*

Q. How ſhall I learn the Addition of *Weight?*

A. Two Sorts of Weight are uſed in *England*, the one call'd *Avoirdupois*, the other *Troy*; both which ought to be underſtood. We ſhall begin with the *Avoirdupois* Weight, as being the moſt common; the ſeveral Diviſions whereof are as follows, and muſt be well remember'd:

AVOIRDUPOIS *Weight.*

16 Drams *make* 1 Ounce.
16 Ounces —— 1 Pound.
28 Pounds —— 1 Quarter of a Hund.
4 Quarters — 1 Hundred.
20 Hundreds — 1 Ton.

In *ſetting down* Sums of this Kind, always begin with the Numbers of the higheſt Denomination, proceeding from the Left Hand to the Right; but in *caſting them up* begin with thoſe of the loweſt Denomination, proceeding from the Right Hand to the Left. As in this Sort of Weight, firſt put down *Tons*, then *Hundreds*, then *Quarters*, &c. but in adding them together do quite the Reverſe, that is, begin with the ſmalleſt Diviſion, and end with the greateſt.

Q. What marks are ſet over the Columns in this Sort of Weight?

A. *T.* is uſed for Tons, *C.* for Hundreds, *qrs.* for Quarters, *lb.* for Pounds, *oz.* for Ounces, and *dr.* for Drams.

Q. In adding up theſe Sums, what is to be carried from one Column to another?

A. For every 16 Drams carry 1 Ounce, for 16 Ounces carry 1 Pound, for 28 Pounds carry 1 Quarter, for 4 Quarters carry 1 Hundred, and for 20 Hundreds carry 1 Ton.

Q. Can't you give me an Example?

A. Yes; Firſt let us ſet down one in *Great Weight*, that is, where the Columns are *Tons*, *Hundreds*, *Quarters*, and *Pounds*.

T.	*C.*	*qrs.*	*lb.*
2	5	1	16
3	4	3	10
1	7	2	03
2	9	1	22
9	7	0	23

Here I begin to caſt up the Column of Pounds, which I find amount to 51, and remembring that for every 28 I muſt carry 1 to the Column of Quarters, I conſider how often 28 is contained in 51; which being but once, and 23 over, I ſet down 23 under the Pounds, and carry 1 to the Quarters. Upon ſumming up theſe I find them to be juſt 8, and as I am to carry 1 for every 4, I put down 0, and carry 2 to the Hundreds. Theſe being added together make 27; and as I am to carry 1 for every 20, I ſet down 7 under the Hundreds, and carry 1 to the Tens; which amounting to 9, I ſet it down underneath, and the Total appears as above.

☞ It may not be amiſs to obſerve, that though ſome Goods are ſold by the Ton, they are not weighed by it, but by Hundreds, Quarters, and Pounds, and the Tons are computed afterwards.

The following is an Example of the *Small Weight*, conſiſting of *Pounds*, *Ounces*, and *Drams*.

lb.	*oz.*	*dr.*
21	11	10
15	08	12
22	12	14
37	09	03
97	10	07

In this Example, remembring that I am to carry 1 for every 16 Drams, and the ſame for every 16 Ounces, I begin with caſting up the Drams, and find they amount to 39, which Number contains twice 16, and 7 over: I therefore ſet down 7 at the Bottom, and carry 2 to the Ounces, which I add together, and find them amount to 42. Now the Number 42 containing twice 16 and 10 over, I put down 10, and carry 2 to the Pounds, which making 97 in all, I ſet down 97, and the Work is finiſhed.

Q. Cannot you give me an Example of the *Great* and *Small* Weight together?

A. Yes; the following may ſerve to employ you when you perfectly underſtand the former.

	(20)	(4)	(28)	(16)	(16)
Tons.	*C.*	*qrs.*	*lb.*	*oz.*	*dr.*
753	19	3	27	15	15
347	08	1	17	10	06
283	11	0	12	03	10
549	05	2	13	09	07
251	13	0	15	03	11
2185	18	1	02	11	01

The Figures at Top, within Parentheſes, are placed there to aſſiſt the Memory, being the Numbers for which 1 is to be carried to the next Column on the Left, as often as they are contained in the Column over which they ſtand. It is not neceſſary to place ſuch a Direction over the laſt Column on the Left, becauſe that is always to be caſt up as Numbers of one Denomination, carrying 1 for every 10 Units to the Place of Tens, and in the ſame Proportion from the Tens to the Hundreds, &c. as was taught in the former Part of *Addition*.—This Method may be obſerved in all Sums of the like Nature, if the Learner wants ſuch Aſſiſtance.

Q. Is there no other Help for the Memory in caſting up large Sums?

A. Yes; in caſting up a long Column or Row of Figures, you may make a Dot with your Pen as often as you come to the Number for which you are to carry 1 to the next Column, and ſet down the Overplus at the Bottom. This is done in the Column of Drams in the laſt Example, where 16 is the Number for which 1 is to be carried to the Column of Ounces. In caſting it up I ſay, 11 and 7 is 18, which being 2 more than 16, I make a Dot againſt the 7, and ſay, 2 and 10 is 12 and 6 is 18; I therefore make another Dot againſt the 6, and carry on the 2, which with 15 makes 17, that is, 1 more than 16: And having no farther to go, I ſet down the Overplus 1 at the Bottom, and for the 3 Dots I carry 3 to the next Column. Thus if you were to caſt up a long Row of Pence, you muſt make a Dot at every 12, and ſetting down the Overplus above the Twelves, carry 1 for each Dot to the Column of Shillings. And this Method may be uſed in all Caſes of the like Nature, wherever it contributes to Eaſe or Expedition.

Q. What Sorts of Goods are weighed by Avoirdupois Weight?

A. All Sorts of Grocery Wares, Butchers Meat, Bread, Butter, Cheeſe, Tallow, Soap, Hemp, Flax, Iron, Tin, Copper, Lead, Pitch, Tar, and other groſs and coarſe Goods in general.

Q. What is the Difference between Troy Weight and Avoirdupois?

A: This will appear by the following Table of

TROY *Weight.*

24 Grains *make* 1 Penny-weight.
20 Penny-weights 1 Ounce.
12 Ounces ——— 1 Pound.

But obſerve, that the *Try* Ounce is ſomewhat larger than the Avoirdupois, 14 Ounces 12 Penny-weights of the former being equal to 16 Ounces of the latter. Take Notice alſo, that Grains are dencted by *grs.* and Penny-weights by *pwts.* or *dwts.* the Pounds and Ounces as before. See an Example.

lb.	*oz.*	*dwts.*	*grs.*
27	11	19	23
39	10	17	19
46	09	13	22
13	10	08	11
158	07	00	03

Q. How do you proceed in caſting up *Troy* Weight?

A. Begin with the Grains, and for every 24 carry 1 to the Penny-weights, for every 20 Penny-weights carry 1 to the Ounces, and for every 12 Ounces carry 1 to the Pounds.

Q. What Things are weighed by *Troy* Weight?

A. Gold, Silver, Jewels, Amber, Electuaries, &c. and alſo Liquors are computed by this Weight, a Pint of Wine, Water, &c. being a Pound.

N. B. Twenty-five Pounds *Troy* is a Quarter of a Hundred. It is likewiſe to be noted, that the Apothecaries make uſe of the *Troy* Pound, Ounce, and Grain, but divide the Ounce into 8 Drams, the Dram into 3 Scruples, and the Scruple into 20 Grains; as in the following Table.

APOTHECARIES *Weight.*

20 Grains *make* 1 Scruple.
3 Scruples —— 1 Dram.
8 Drams —— 1 Ounce.
12 Ounces —— 1 Pound.

Here obſerve, that though Apothecaries compound their Medicines by this Weight, they buy and ſell their Drugs by Avoirdupois.

Q. How do you carry in caſting up Sums of this Weight?

A. For every 20 Grains carry 1 to the Scruples, for every 3 Scruples carry 1 to the Drams, for every 8 Drams carry 1 to the Ounces, and for every 12 Ounces carry 1 to the Pounds.

Q. What are the *Meaſures* uſed in *England?*

A. There are various Sorts, as you will learn from the enſuing Tables.

WINE

WINE *Meaſure.*

2	Pints *make*	1	Quart,
4	Quarts —	1	Gallon.
63	Gallons —	1	Hogſhead.
2	Hogſheads	1	Pipe or Butt.
2	Pipes ——	1	Tun.

In adding this Meaſure, for every 4 Quarts carry 1 to the Gallons, for every 63 Gallons 1 to the Hogſheads, for every two Hogſheads 1 to the Pipes, and for every 2 Pipes 1 to the Tuns.——*N. B.* The Wine Gallon contains 231 cubic or ſolid Inches, by which all Liquids are meaſured, except Beer and Ale.

BEER *and* ALE *Meaſure.*

2	Pints *make*	1	Quart.
4	Quarts	1	Gallon.
9	Gallons	1	Firkin.
2	Firkins	1	Kilderkin.
2	Kilderkins	1	Barrel.
3	Barrels	1	Butt.

The Beer and Ale Gallon are the ſame, containing 282 ſolid Inches; but take Notice, that 8 Gallons make a Firkin of Ale, and conſequently a Barrel of Ale is 4 Gallons leſs than a Barrel of Beer. In adding this Meaſure, for every 9 Gallons carry 1 to the Firkins, and for every 4 Firkins carry 1 to the Barrels. N. B. One Barrel and a half, or 54 Gallons, is a Hogſhead of Beer.

DRY *Meaſure.*

8	Pints *make*	1	Gallon.
2	Gallons —	1	Peck.
4	Pecks —	1	Buſhel.
4	Buſhels —	1	Coom.
2	Cooms —	1	Quarter.
5	Quarters —	1	Whey.
2	Wheys —	1	Laſt.

N. B. Four Quarters are commonly called a Chaldron, but in *London* the Chaldron of Coals is 36 Buſhels. In adding, for every 2 Gallons carry 1 to the Pecks, for every 4 Pecks carry 1 to the Buſhels, for every 8 Buſhels 1 to the Quarters, &c.

LONG *Meaſure.*

3	Barley-Corns	*make*	1 Inch.
12	Inches	——	1 Foot.
3	Feet	——	1 Yard.
5	Yards and 1-half		1 Pole or Perch
40	Poles	——	1 Furlong.
8	Furlongs	——	1 Mile.

In adding this Meaſure, for every 12 Inches carry 1 to the Feet, for every 3 Feet 1 to the Yards, &c. N. B. Yards are ſometimes ſet down in Half-Yards, and Feet in Inches; and then for every 18 Inches, 1 is carried to the Half-Yards, and for every 11 Half-Yards 1 to the Poles.

LAND

LAND or SQUARE *Meaſure.*

40	Square Poles	*make*	1 Rood.
4	Roods	——	1 Acre.

In this Meaſure carry 1 for every 40 Poles, and 1 for every 4 Roods.

CLOTH *Meaſure.*

4	Nails	*make*	1 Quarter.
4	Quarters	—	1 Yard.
5	Quarters	—	1 Ell *Engliſh.*
3	Quarters	—	1 Ell *Flemiſh.*

In adding, for every four Nails carry 1 to the Quarters, for every 4 Quarters carry 1 to the Yards; for every 5 Quarters carry 1 to the Ells if *Engliſh*, and for every 3 if *Flemiſh.*

TIME.

60	Seconds	*make*	1 Minute.
60	Minutes	——	1 Hour.
24	Hours	——	1 Natural Day
7	Days	——	1 Week.
4	Weeks	——	1 Month.
13	Months, 1 Day and 6 Hours	}	1 Solar Year

But the Year is commonly divided into 12 Calendar Months, ſome containing 30 Days, others 31, amounting in all to 365. *Note*, however, that the true Solar Year is equal to 365 Days, 5 Hours, 49 Minutes, 4 Seconds, and 21 Thirds. In the Addition of Time, for every 60 Seconds carry 1 to the Minutes, for every 60 Minutes carry 1 to the Hours, &c.

I think it needleſs to ſet down Examples of theſe ſeveral Meaſures, for if thoſe of *Money* and *Weights* be well underſtood, and theſe Tables carefully attended to, the Scholar will ſoon know how to *carry* from one Denomination to another, and conſequently be able to caſt up any Sum whatſoever in *Compound Addition*. I ſhall therefore only give the Proof of this Rule, and proceed to *Subtraction*.

Proof of Addition.

The beſt Way of proving *Addi-*
on, whether *Simple* or *Compound,*
to begin at the Top of the Sum,
nd reckon the Figures downwards,
the ſame Manner that they were
dded upwards ; and if the ſecond
otal be the ſame as the firſt, the
peration has been performed right,
therwiſe there is ſome Miſtake.

CHAP. III.

Of SUBTRACTION.

Q. WHAT is SUBTRACTION ?

A. It is *the taking a leſſer*
umber from a greater, in order to find
e Difference between them.

Q. What is to be obſerved in pre-
aring Numbers for this Operation ?

A. Set down the leſſer Number un-
er the greater, and let Units ſtand

under Units, Tens under Tens, &c. in the ſame Manner as if they were prepared for Addition.

Q. What is the next Thing to be done?

A. Having drawn a Line underneath the Numbers, begin at the lower Figure in the firſt Column to the Right Hand, and ſubtract it from the upper one, ſetting down the Remainder under the Line; and ſo proceed towards the Left, ſubtracting every lower Figure in each Column from that above it, *i. e.* Units from Units, Tens from Tens, Hundreds from Hundreds, &c. and putting the ſeveral Remainders under the Line in their correſpondent Places.

Q. Is there nothing farther to be obſerved in ſubtracting?

A. When the upper Figure is leſs than the lower, and conſequently the lower

ower cannot be ſubſtracted from it, add 10 to it, which is called *borrowing* ſo many from the next ſignificant Figure on the Left Hand in the upper Line; and for the 10 ſo borrowed you muſt pay 1 to the next Figure or Cypher on the Left Hand in the lower Line: As will be made plainer by Examples.

Suppoſe I buy 325 Pens, and ſell 236, how many ſhall I have remaining?

	Pens.
Bought	325
Sold -	236
Remain	089

Here I begin with the loweſt Figure in the firſt Column on the Right Hand, ſaying, 6 from 5 I cannot take, but (borrowing 10 and putting it to the 5) 6 from 15 and there remains 9, which I ſet down underneath, and for the 10 that I borrowed and added to the 5 in the upper Line, I add 1 to the 3 in the lower Line, ſaying, 1 that I borrowed and 3 is 4; but not being able to take 4 from 2, I again borrow 10, and ſay, 4 from 12 and there remains 8, which I ſet down; and for the 10 that I borrowed I add 1 to the 2, ſaying 3 from 3 and there remains 0, which I put down under the laſt Column, and find the Remainder to be 89, as you ſee in the Example.

Again: Let it be required to take the Number 3082 from 4205, I ſet down the Figures as above directed.

From 4205
Take 3082

Rem. 1123

In this Example, beginning with the Column of Units as uſual, I ſay, 2 from 5 and there remains 3, which I place underneath and proceed; 8 from 0 I cannot, but 8 from 10 (which I borrow) and there remains 2: This being ſet down, I ſay, 1 that I borrowed and 0 is 1, which take from 2 and there remains 1; then 3 from 4, and 1 alſo remains. Thus I find the Difference between the two given Numbers to be 1123.

The Number of Years elapſed ſince any Year paſt may be known by ſubtracting the Date of the paſt Year from the Date of the preſent. Suppoſe then I would know how many Years it is ſince the Gun-powder Plot, which happened in the Year 1605, I ſet down the Date of the preſent Year 1788 and the other under it, and then ſubtracting in the Manner already ſhewn, I find the Difference to be 183, the Number of Years required.

Preſent Year	1788
Year of the Plot	1605
Years ſince	183

Q. How do you proceed in ſubtracting Numbers of divers Denominations?

A. Obſerve this general Rule, that whatever Number you *carried for* in Addition, the ſame Number you muſt *borrow* in Subtraction, if it be wanted. For inſtance, in adding Pence together, for every 12 you carried 1 to the Shillings; and ſo in ſubtracting Pence from Pence you muſt borrow 12 from the Shillings, that is, 12 Pence, when Occaſion requires. And this you muſt always pay to the next Place on the ~~Left, by~~ adding 1 to the Figure in the lower Line. Here follows an Example in the Subtraction of Money.

	l.	*s.*	*d.*
Borrowed	242—	16—	3¼
Paid	174—	17—	9½
Remains to pay	67—	18—	5¾

In this Example I begin with Farthings, ſaying, 2 from 1 I cannot take, but borrowing a Penny, that is 4 Farthings, and adding it to the 1, I ſay, 2 from 5 and there remains 3 Farthings, which I ſet down thus ¾, as you ſee above. Then for the 4 Farthings that I borrowed I add 1 to the 9 in the Column of Pence, ſaying 1 and 9 is ten, which I cannot take from 3; but borrowing 12 Pence, (*i.e.* a Shilling) and adding them to the 3, I ſay, 10 from 15 and there remains 5, which I put down under the Pence, and proceed, ſaying, 1 that I borrowed (that is, 1 Shilling) and

and 17 is 18, 18 from 16 I cannot, but borrowing 20 Shillings, (or 1 Pound) and adding them to the 16, which make 36, I ſay 18 from 36, and there remains 18; this I ſet down under the Shillings, and for the 20 Shillings borrowed I add 1 to the Pounds, ſaying, 1 and 4 is 5, 5 from 2 I cannot, but borrowing 10 from the next Place, and adding it to the 2, I ſay, 5 from 12 and there remains 7, which I put down under the Units, and for the 10 I borrowed, I add 1 to the 7 in the lower Line, ſaying, 8 from 4 I cannot, but 8 from 14 (borrowing 10 more) and there remains 6; then 1 for the 10 I borrowed and 1 is 2, 2 from 2 and there remains nothing. Thus the Work is compleated, and the Money ſtill to be paid appears to be *Sixty-ſeven Pounds, eighteen Shillings, and five Pence three Farthings.* N. B. In ſubtracting the Pounds you borrow by Tens, as in Numbers of one Denomination; and ſuch the laſt Column on the Left Hand is always to be reckoned, let its Name be what it will.

Another Example.

	l.	*s.*	*d.*
From	1296	10	10½
Take	173	5	7¼
Rem.	423	5	3¼

Q. What is to be obſerved in the Subtraction of *Weight?*

A. In the Subtraction of *Avoirdupois* Weight, in Caſe of Want in the Drams or Ounces borrow 16, in the Pounds 28, and in the Quarters 4. In *Troy* Weight, when Neceſſity requires, borrow 24 in the Grains, 20 in the Penny-weights, and 12 in the Ounces; always remembring to pay 1 for what you borrow to the next Place on the Left Hand in the lower Line. And you may ſet theſe Numbers over the Columns to aſſiſt your Memory if you think proper, as you ſee in the following Examples:

AVOIRDUPOIS.

		(4)	(28)	(16)
	C.	*qrs.*	*lb.*	*oz.*
From	16	3	13	9
Take	8	1	21	12
Rem.	8	1	19	13

Here, beginning with the Ounces, I ſay, 12 from 9 I cannot, but 12 from 16 and there remains 4, which added to 9 makes 13. I therefore ſet down 13 under the Ounces, and for the 16 that I borrowed I add 1 to the Pounds, ſaying, 22 from 13 I cannot, but 22 from 28, and there remains 6, which with 13 makes 19. I then ſet down 19, and for the 28 that I borrowed, I add 1 to the Pounds, ſaying 1 and 1 is 2; 2 from 3 and there remains 1; which being put down under the Quarters I proceed, ſaying, 8 from 16 and there remains 8: And this being ſet down under the Hundreds the Operation is finiſhed.

TROY.

	(12)	(20)	(24)
	lb.	*oz.*	*dwt.*	*grs.*

	lb. (12)	*oz.* (20)	*dwt.* (24)	*grs.*

From 62—4—10—11
Take 41—9—06—16

20—7—03—19

In this Example I ſay 16 from 11 I cannot take, but 16 from 24 and there remains 8, which added to 11 makes 19. Then ſetting down 19 under the Grains, for 24 I borrowed, I add

I add 1 to the Penny-weights in the lower Line, ſaying 1 and 6 is 7, which take from 10 and there remains 3. I then put down 3 and proceed, ſaying, 9 from 4 I cannot, but 9 from 12 and there remains 3 and 4 is 7; which being ſet down I carry 1 to the Pounds, ſaying, 1 and 1 is 2, 2 from 2 and there remains 0, 4 from 6, and there remains 2. Thus the Work is compleated, and ſtands as you ſee above.

Q. How do you proceed in the Subtraction of *Meaſure?*

A. In the ſame Manner as you have ſeen in the Examples of *Weights* and *Money*; for in all Kinds of *Compound* Subtraction the chief Buſineſs is to remember, that the Number borrowed in any Column is always ſo many as will make 1 in the next; and how many of a lower Denomination make one of a higher may be learnt from the foregoing Tables.

Q. How do you prove that your Work is performed right?

A. The Proof of Subtraction is very eaſy; namely, by adding the Remainder to the Sum ſubtracted, and, if there be no Miſtake, the Total will be the ſame with the uppermoſt Line, from whence the Subtraction was made; as appears in the following Inſtance.

l. *s.* *d.*
From 42—16—09
Take 18—16—11 } Add
Rem. 23—19—10 }
Proof 42—16—09

I ſhall conclude this Rule with one Example, from whence the Learner may diſcern how all Queſtions of the ſame Nature may be anſwered. *Suppoſe I have already ſaved up 39l. 18s. and 10d. how much Money do I want to make up a Hundred Pounds?*

	l.	s.	d.
From	100	00	00
Take	39	18	10
Anſwer	60	01	02

CHAP. IV.

Of MULTIPLICATION.

Q. WHAT is MULTIPLICATION?

A. It is a Rule whereby *from two Numbers given we find out a third, which contains either of the two Numbers as often as the other contains Units.* Or, it is the finding what will be *the Sum of any Number added to itſelf,* or repeated *as often as there are Units in* n *another*; being a compendious Kind f Addition.

Q. What is to be obſerved in order o underſtand this Rule ?

A. Take particular Notice, that of he two given Numbers the one is alled the *Multiplicand*, which is the Number to be multiplied; the other he *Multiplier*, by which we multiply; nd the third Number, ariſing from he Operation is called the *Product*, which is the Multiplicand ſo often added to itſelf as there are Units in he Multiplier. But before any Progreſs can be made in Multiplication, it is neceſſary for the Learner to get the following Table by Heart, and that very perfectly.

MULTIPLICATION TABLE.

3 times	3 is	9
	4	12
	5	15
	6	18
	7	21
	8	24
	9	27
4 times	4	16
	5	20
	6	24
	7	28
	8	32
	9	36
5 times	5	25
	6	30
	7	35
	8	40
	9	45
6 times	6 is	36
	7	42
	8	48
	9	54
7 times	7	49
	8	56
	9	63
8 times	8	64
	9	72
9 times	9	81
12 times	2	24
	3	36
	4	48
	5	60
	6	72
	7	84
	8	96
	9	108

When this Table is perfectly learn'd by Heart, ſo as to know the Product of any two Figures multiplied one by another without Heſitation, you may proceed to work any Sum propoſed.

Q. How are Numbers to be ſet down in order for Multiplication?

A. Firſt ſet down the *Multiplicand*, which is generally the greater of the given Numbers, and then the *Multiplier* underneath it, Units under Units, Tens under Tens, &c.

Q. How do you proceed in the Operation?

A. Having drawn a Line under the Multiplier, begin with the Figure in the Units Place, and with that multiply the Figure in the ſame Place of the Multiplicand, ſetting down the Product, if it be leſs than 10, under the Line in the Place of Units; but if the Product be more than 10, (or any Number of Tens) ſet down the

Overplus, as in Addition, and carry the Ten or Tens in Mind till you have multiplied the next Figure of the Multiplicand with the ſame Figure of the Multiplier, and to their Product add 1 for every 10 in the former Product, ſetting down the Overplus above the Tens as before; and ſo proceed till you come to the laſt Figure of the Multiplicand, where the whole Product muſt be ſet down, becauſe the Tens can be carried no farther.---But an Example will make this plainer.

Let it be required to multiply 1745 by 6, and the Work will ſtand thus:

Multiply 1745 *Multiplicand.*
by 6 *Multiplier.*

10470 *Product.*

Here I begin with the Units, as above directed, ſaying, 6 times 5 is

30

30, which being juſt 3 Tens and nothing under nor over, I ſet down 0, and carry 3 to the next Place, ſaying, 6 times 4 is 24 and 3 is 27, which being 7 more then 2 Tens or 20, I put down 7 and carry 2. Then I ſay, 6 times 7 is 42, and 2 is 44, which Number containing 4 Tens and 4 over, I ſet down 4 and carry 4, ſaying, 6 times 1 is 6 and 4 is 10, which I ſet down; having gone through all the Figures of the Multiplicand, and the Product appears to be 10470, as you ſee above.

Q. But how do you proceed when there are more than one Figure in the Multiplier?

A. When the Multiplier conſiſts of two or more Figures, begin with that in the Units Place, as before; and having gone through the Multiplicand, and ſet down the Product under the Line, do the ſame by the Figure in the Tens Place, ſetting down this Product one Remove nearer to the Left Hand, that is, its firſt Figure under the ſecond of the former Product, the ſecond under the third, &c. The ſame is to be obſerved in ſetting down the Product of each Figure of the Multiplier; and then drawing a Line under the ſeveral Products, and adding them together, you have the general Product required.

Example.

Multiply 1745
by 36

10470
5235

Gen. Product 62820

Here the Product by 6 being found and ſet down as in the former Example, I proceed to multiply by 3, ſaying, 3 times 5 is 15, that is, 5 more than 10, therefore I ſet down 5, the firſt Figure of this Product, under 7, which ſtands in the ſecond Place of the former Product, as above directed. Then I ſay, 3 times 4 is 12, and 1 that I carry is 13, ſet down 3 and carry 1; then 3 times 7 is 21 and 1 is 22, ſet down 2 and carry 2; laſtly, 3 times 1 is 3 and 2 is 5, which I put down, and the Multiplication is finiſhed; and by adding the two Products together the general one is found to be as above expreſſed.— To make this plainer take another Example: Multiply 13326

by 4839

119934

39978

106608

53304

64484514

7

Q. What is to be done when Cyphers are intermixed with Figures in the *Multiplier?*

A. Proceeding with the Figures as above directed, when you come to a Cypher in the Multiplier, inſtead of making a Line of Cyphers among the other Products, ſet down one Cypher directly under that in the Multiplier, and begin the Multiplicand again with the next Figure, putting down the Product in the ſame Line with the Cypher: But obſerve, that the firſt Figure, or Cypher, of the next Product (if ſuch there be) muſt be ſet down a Degree farther towards the Left Hand, not immediately under the Figure that ſtands next to the Cypher above; as may be underſtood by the ſecond of the following Examples.

Example 1.

4392
403

13176
175680

1769976

Example 2.

62725
2307

439075
1871750
125450

144606575

N. B. It will be the ſame Thing, when you come to a Cypher in the Multiplier, if you leave its Place vacant in the Product, or two Places if there be two Cyphers together, *&c.* proceeding to multiply by the next ſignificant Figure, and always remembring to put down the firſt Figure or Cypher of a Product perpendicularly under the Figure by which you multiply.

Q. Is there not a ſhort Way of multiplying when you have Cyphers on the Right Hand of the given Numbers ?

A. Yes ; when either your Multiplicand, or Multiplier, or both, have one or more Cyphers to the Right Hand, only multiply by the ſignificant Figures, and then put the Cyphers contained in both or either of them to the Right Hand of the Product ; as in the following Examples.

200	232	4000
3	200	3000
600	46400	12000000

In the two firſt Examples the Multiplicand and Multiplier are not ſet down according to the general Rule, that the Work may appear the plainer; but if they had been placed as uſual, the Product would have been the ſame.

N. B. When you are to multiply

any Number by 10, 100, 1000, &c. it is only annexing one, two, three, or more Cyphers to the Multiplicand, and the Work is done. Suppoſe, for Inſtance, I would multiply 275 by 10, the Addition of one Cypher gives me the Product, viz. 2750; if I multiply by 100, I add two Cyphers, and it makes 27500, &c.

There is alſo a ſhort Way of multiplying by 5, namely, to add a Cypher to the Multiplicand and then halve it, which Half is the Product. For Example, if I would multiply 244 by 5, by annexing a Cypher it becomes 2440, the Half whereof is 1220, which is the Product required.

Q. How do you multiply Numbers of divers Denominations?

A. Begin with the leaſt, and ſo proceed from one to another till you come to the greateſt, carrying from the lower Denomination to the higher, as you do in Addition.---Let us firſt give an Example in the Multiplication of Money.

Suppoſe I have three Purſes, and 9*l.* 3*s.* 7*d.* in each of them, how much does the Whole amount to?

This might be known by Addition, that is, by ſetting down the given Sum 3 Times, and then adding them all together; but all ſuch Queſtions as theſe are much more expeditiouſly anſwered by Multiplication. I therefore put down the Sum as follows:

	l.	*s.*	*d.*
Multiply	9—	3—	7
by			3
Anſwer	27—	10—	9

Here I ſay, 3 times 7 is 21, which being 9 more than 12, that is Ninepence above a Shilling, I ſet down 9 under the Pence, and carry 1 to the

Shillings, ſaying, 3 times 3 is 9 and 1 is 10, which being leſs than a Pound I have therefore none to carry, but put down 10 under the Shillings; then I ſay, 3 times 9 is 27, which I put down under the Pounds, and the Work is compleated.

Again: If one Piece of Cloth coſt 7*l.* 9*s.* 6*d*½, what will be the Price of eight Pieces?

	l.	*s.*	*d.*
Multiply	7—	9—	6
by			8
Anſw.	59—	16	4.

In this Example I begin with the Halfpenny, ſaying, 8 Half-pence make 4 Pence, which I carry to the Pence, and ſay, 8 times 6 is 48 and 4 is 52; which being 4 Pence above 4 Shillings, I ſet down 4 under the Pence, and carry 4 to the Shillings, ſaying, 8 times 9 is 72 and 4 is 76, which is equal to 3 Pounds 16 Shillings; I therefore put down 16, and carry 3 to the Pounds, ſaying, 8 times 7 is 56 and 3 is 59; which being ſet down, as above, we have the Anſwer to the Queſtion.

Q. How do you proceed when the Multiplier conſiſts of two Figures?

A. If the Multiplier be above 12, the Queſtion is ſometimes anſwered by two Multiplications, and ſometimes by three, and adding the Product of the laſt to the Product reſulting from the two former.—Suppoſe it be aſked, *What* 15 *Yards of Lace come to at* 1l. 3s. 6d. *per Yard?* Here I conſider, that as 3 times 5 makes 15, I can perform this Work in two Multiplications, *viz.* by 3 and 5; as in the following Example.

	l.	*s.*	*d.*	
Multiply	1	3	6	
by			3	
Multiply	3	10	6	Pr. of 3.
by			5	
Anſwer	17	12	6	Pr. of 15

This Example, I apprehend, needs no particular Explanation; for the given Price being firſt multiplied by 3, and then the Product of that by 5, the Reſult of theſe two Multiplications is plainly the Anſwer required.---But if it be aſked, *What is the Product of* 2l. 13s. 4d. *multiplied by* 39? This Queſtion cannot be anſwered by two Multiplications like the former, becauſe there are no two Numbers which multiplied together will produce 39 exactly. Endeavouring however to come as near to it as poſſible, I conſider that 4 times 9 make 36, and therefore I make uſe of 4 and 9 as Multipliers in this Operation. Having thus found the Amount of the given Sum multiplied by 36, I multiply it by 3, which makes 39, and the two Products added together give the Anſwer to the Queſtion ; as you may underſtand by a View of the Work.

	l. *s.* *d.*	
Mult.	2—13—4	
by	4	
And	10--13--4	Pr. of 4.
by	9	
Add	96--00--0	Pr. of 36
	8--00--0	Pr. of 3
Anſw.	104--00--0	

I forbear giving more Examples of this Nature, for if theſe be well underſtood it will be no great Difficulty to multiply by any Number whatſoever, obſerving the ſame Method.—But it may not be amiſs to ſet down another Example, in order to ſhew the Scholar one conſiderable Uſe of Multiplication, namely, its bringing great Denominations into ſmall, as Pounds into Shillings, Pence, or Farthings. Suppoſe, for Inſtance, I would know how many Shillings there are in 36*l.* I need only multiply 36 by 20, (becauſe 20*s.* make 1 Pound) and the Product gives the Number of Shillings required.

	l.
Multiply	36
	20
Prod.	720 Shillings.

Now if 720 were multiplied by 12, (there being ſo many Pence in a Shilling) the Product would give the Number of Pence; and if that Product were again multiplied by 4, (the Farthings in a Penny) it would ſhew the Number of Farthings contained in 36 Pounds.---But more of this when we come to ſpeak of *Reduction.*

Q. In what Manner is *Multiplication* to be proved?

A. There are ſeveral Ways of proving it, firſt by Diviſion, for when the Product divided by the Multiplier quotes the Multiplicand, or divided by the Multiplicand quotes the Multiplier, the Work is certainly right: But the Scholar having not yet learnt Diviſion, the bare mention of this is ſufficient. Another Way is to caſt the Nines out of the Multiplicand, Multiplier and Product; but this Method being erroneous, we recommend the following, *viz.* make that which was your Multiplicand your Multiplier, and that which was your Multiplier your Multiplicand, and the Product will be the ſame if the Work be truly performed, as will appear by ſetting down one Example.

Multiply	243	176
by	176	243
	1458	528
	1701	704
	243	352
	42768	42768

CHAP. V.

Of DIVISION.

Q. WHAT is DIVISION?

A. It is *a Rule whereby we diſcover how often one Number is contained in another.* As Multiplication is a compendious Way of Addition, ſo Diviſion ſupplies the Place of many Subtractions.

Q. What are the Terms made uſe of in this Rule?

A. They are four, namely, the *Dividend*, or Number to be divided; the *Diviſor*, or Number by which we divide; the *Quotient*, or Number ariſing from the Diviſion, which ſhews how often the Diviſor is contained in the Dividend; and laſtly the *Remainder*, which is the Number left out of the Dividend after the Diviſor has been taken from it as often as is expreſſed in the Quotient; but this is merely accidental, for ſometimes there is no Remainder at all.

N. B. The Remainder muſt always be leſs than the Diviſor.

Q. Are there not two Sorts of Diviſion?

A. Yes, *Single* and *Compound*.

Q. What is *Single* Diviſion?

A. It is called *Single* when the Diviſor is but one Figure, and the Dividend but two at moſt. --- Any Queſtion of this Kind is anſwered by the Multiplication Table; as if 63 were to be divided by 7, the Anſwer will be 9. Here 63 is the *Dividend*, 7 the *Diviſor*, and 9 the *Quotient*, ſhewing that the Number 7 is contained 9 times in the Number 63.

Q. What is *Compound* Diviſion?

A. It is called *Compound* when the Dividend conſiſts of more Figures or Cyphers than two, and the Diviſor of one or more Figures or Cyphers.

Q. How do you ſet down a Sum in order to be divided?

A. Firſt write down your Dividend, ſuppoſe it to be 365, and on the Left Hand of it your Diviſor, ſuppoſe 7, ſeparating them by a crooked Line, or *Parentheſis*; then make another crooked Line on the Right of the Dividend, where you muſt place your Quotient when found; and the Work will ſtand as follows:

Divid.
Diviſor 7) 365 (

Q. How do you proceed in the Operation?

A. As I cannot take 7 out of 3, the firſt Figure of the Dividend, I therefore reckon the firſt and ſecond together, and conſider how often I can take 7 out of 36; which being 5 times, I ſet down 5 in the Place appointed for the Quotient, and with that I multiply the Diviſor 7, ſaying 5 times 7 is 35, which I put down exactly under 36 the two firſt Figures of the Dividend: Then drawing a Line I ſubtract 35 from 36, and there remains 1, to which bring down the laſt Figure of the Dividend, *viz.* 5, and then there is 15 for a new Dividend, or Dividual, to work upon. I therefore conſider again, how often 7 may be had out of 15, which being twice, I place 2 in the Quotient next to the 5, and again multiply the Diviſor by 2, ſaying twice 7 is 14, which I ſet down under the 15, from whence I ſubtract it, and find 1 to be the Remainder; as will be better underſtood by a View of the Work at length.

Divid.

Diviſor 7)365(52 *Quotient.*

35

15

14

---

1 *Remainder.*

Thus I find that the Number 7 is contained 52 times in 365, beſides which there is 1 remaining: And this Operation is the ſame as if I had aſked how many Weeks there are in a Year; for there being 365 Days in a Year, and 7 Days in a Week, the Number 365 divided by 7 ſhews that there are 52 Weeks, and 1 Day over.

N. B. If there had been more Figures or Cyphers in the Dividend, they muſt all have been brought down, one by one, (and never more than one at a time) and placed with the remaining Number after Subtraction, as the 5 was brought down and added to the 1 in the Example above. You are alſo to obſerve, that for every Figure or Cypher brought down from the Dividend in order for a new Operation, there muſt be a Figure or a Cypher ſet down in the Quotient. And that you may the better remember what Figures or Cyphers have been ſo brought down and done with, you may make a Dot under them in the Dividend, as you ſee in the following Example.

Suppoſe a Father dies, and leaves 8440*l*, to be equally divided amongſt

8 Children, how much will be the Share of each?

```
8) 8440 (1055
   8...
   ----
   4
   0
   ----
   44
   40
   ----
    40
    40
   ----
     0
   ----
```

Here I begin by enquiring how many times the Diviſor 8 can be had out of 8 the firſt Figure in the Dividend, which being but once, I ſet down in 1 the Quotient, and 8 under 8 in the Dividend, from whence I ſubtract it and there remains 0; but a

Cypher being of no Value on the Left Hand of a Figure, I do not put it down, but leave the Place vacant, then bringing down the 4 from the Dividend, and making a Dot under it to ſhew that I have done ſo, I ſay, 8 from 4 cannot be had, therefore I put a Cypher in the Quotient; and as I cannot multiply the Diviſor by Nothing, I alſo put down 0 under the 4, and proceed to Subtraction, ſaying, 0 from 4 and there remains 4; to which I bring down the next 4 in the Dividend, making a Dot under it as before, and this gives me a new Dividual, *viz.* 44. I then conſider, that 8 may be had 5 times out of 44, and accordingly put down 5 in the Quotient; then multiplying 8 by 5, which makes 40, I place 40 under the 44, and ſubtract it as uſual: To the remaining 4 I then bring down the 0 from the Dividend, which making 40, I conſider

ſider again how many Eights that Number contains, and finding them to be 5 I ſet down another 5 in the Quotient, by which I likewiſe multiply the Diviſor, ſaying, 5 times 8 is 40; and this being put down and ſubtracted from the other 40, the Work is compleated, and nothing remains. The Quotient therefore, 1055, gives the Anſwer to the Queſtion, for juſt that Number of Pounds is the Share of each Child.

Q. Is there no other Method of working in Diviſion than this?

A. Yes; when the Diviſor conſiſts only of one Figure, as in the Examples above, the Work may be performed in a much more compendious Manner, and which in my Opinion will be more eaſy to the Learner. In this Method you are to draw a Line under the Dividend, and ſet down under its firſt Figure how often the Diviſor is contained in it; if any Thing remains, imagine it placed before the next Figure, and conſider how often the Diviſor is contained in the Sum it makes: ſet down the Number underneath as before; and ſo proceeding through all the Figures of the Dividend, put down what remains at laſt where the Quotient ſtands in the former Examples.—An Inſtance or two will make this plain.

Let it be required to divide 78906 by 4, that is, into four equal Parts; the Work when finiſhed, will ſtand thus:

Dividend

Diviſor 4)78906(2 *Remainder.*

Quotient 19726

In working this Example I ſay, it may be had once in 7, and there remains 3; I therefore ſet down 1 under the Line, and ſuppoſe the 3 to be placed before 8, the next Figure in the Dividend, which makes it 38. I then proceed to enquire how many times 4 in 38, which being 9 times and 2 over, I ſet down 9 in the Quotient, and carry 2 to the 9 in the Dividend, which makes it 29; then the Fours in 29 are 7, and 1 over, therefore I put down 7, and ſuppoſing the 1 to be placed before the 0 in the Dividend, which would make 10, I proceed to conſider how many times 4 is contained in that Number; which being twice, and 2 over, I ſet down 2 in the Quotient, and carrying the Overplus 2 to the 6 in the Dividend I call it 26; then laſtly I enquire how many times 4 is contained in 26, which being 6 times and 2 over, I put down 6 under the Line, and the remaining 2 on the Right Hand of the Dividend, and the

Operation is compleated.—I ſhall ſet down one more Example for the Learner's Practice.

$$\begin{array}{c}6)479620(4\\ \hline 79936\\ \hline\end{array}$$

N. B. This is the ſhorteſt Way of Diviſion that can be by a ſingle Figure; and as it is a Halving, Thirding, Fourthing, *&c.* of the Dividend, ſome chuſe to expreſs the Diviſor in the Manner of a Fraction, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, *&c.* as in the following Examples:

$$\frac{1}{3}\begin{array}{c}23412\\ \hline 7804\\ \hline\end{array}\qquad\qquad \frac{1}{7}\begin{array}{c}42721\\ \hline 6103\\ \hline\end{array}$$

In this Caſe the Method of working is the ſame as above, ſaying, the Third of 23 is 7 and 2 over, which 2 annexed to the 4 makes 24; then the Third of 24 is 8; the Third of 1 cannot be had, therefore put down a Cypher; but the 1 reckoned with the 2 makes 12, the Third of which is 4 without any Remainder, therefore ſet down 4 in the Quotient, and the Work is finiſhed.—And thus Diviſion by any ſingle Figure is readily performed.

Q. But how do you proceed when the Diviſor conſiſts of two or more Figures?

A. The working in this Sort of Diviſion is not quite ſo eaſy, but I ſhall endeavour to make it as plain as the Nature of the Thing will admit. —Suppoſe then that 352 Oranges were to be equally divided amongſt 32 Boys, how many would each Boy have to his Share?

Here 352 is the Dividend, and 32 the Diviſor, which I ſet down in the uſual Manner, and having gone thro the Operation as explained below, the whole Work ſtands as follows:

```
32)352(11
   32
   ——
    32
    ——
     0
    ——
```

Firſt I enquire how many times 32 is contained in 35, the two firſt Figures of the Dividend, which I find to be once; and it would have been the ſame Thing if I had aſked how often 3, the firſt Figure of the Diviſor, was contained in 3 the firſt of the Dividend: I therefore put down 1 in the Place of the Quotient, ſaying, once 32 is 32, which I place under 35 in the Dividend; and ſubtracting it from thence there remains 3: To this I bring down 2 from the Dividend, and theſe give me a new Dividual, namely 32, which being the ſame as the Diviſor, muſt neceſſarily contain it once and no more; I therefore ſet down another 1 in the Quotient, and alſo 32 under 32 the laſt Dividual, from whence I ſubtract it, and nothing remains.—And here the Quotient ſhews how many Oranges each Boy muſt have, namely *eleven*.

Let us make another Trial, and divide 1231 by 36, which is the ſame Thing as if I ſhould aſk how many Yards there are in 1231 Inches.

Having ſet down the Diviſor and Dividend, 36)1231, I am ſenſible at firſt Sight that the two Figures of which the Diviſor conſiſts cannot be taken from the two firſt Figures of the Dividend, and therefore I muſt have three, and enquire how often 36 may be had in 123; but as this is not readily anſwered by a young Practitioner in Diviſion, I ſhall go an eaſier Way to work, (as may always be done where the Diviſor conſiſts of ſeveral Figures) and try how often the firſt Figure of the Diviſor may be had in the firſt of the Dividend; but as in this Example it cannot be had at all, I take two Figures of the Dividend, and aſk how many times 3 in 12. The Anſwer is plainly 4 times; but before I ſet down 4 in the Quotient, I multiply the Diviſor by it, (either in my own Mind, or upon ſome waſte Piece of Paper) to ſee whether the Product will not amount to more than the three firſt Figures of the Dividend, *viz.* 123, from whence it is to be ſubtracted; and finding that 4 times 36 is 144, which cannot be taken from 123, I only put down 3 in the Quotient, and then multiply the Diviſor by it, placing the Product, *viz.* 108, under 123 in the Dividend, and after Subtraction, there remains 15, the Work ſtanding thus:

```
36)1231(3
   108
   ———
    15
```

To this 15 I bring down 1, the next and laſt Figure of the Dividend, which gives me 151 for a new Dividual. Then, inſtead of aſking how many times 36 in 151, I enquire (as before) how often 3 in 15, which I readily ſee to be 5 times; but multiplying the Diviſor by 5, I find the Product would again be too large for Subtraction, and therefore I ſet down but 4 in the Quotient, by which I multiply the Diviſor, and the Product is 144. This being placed under the Dividual 151, and ſubtracted from it, I find 7 remaining; and there being no more Figures to bring down to it from the Dividend, the Work is finiſhed, and appears as follows:

36)1231(34$\frac{7}{36}$
108
———
151
144
———
7
———

Hence I learn, that in 1231 Inches there are 34 Yards and 7 Inches, or 7 Parts of a Yard out of 36; for the Remainder is always a Fraction, and is frequently placed over the Diviſor on the Right Hand of the Quotient, as in the Example above.

In the ſame Manner any larger Number may be divided; but when the Diviſor conſiſts of many Figures, it is a great Eaſe to the Memory to make a Table of it, multiplying it ſeverally by all the nine Digits, and ſetting down the Numbers in the following Order.—Let 642 be the Diviſor, which muſt be placed over againſt the 1; then multiply 642 by 2, and oppoſite to it ſet down the Product; then multiply 642 by 3, placing the Product over againſt it, and ſo proceed till you have finiſh'd the Table.

1	642
2	1204
3	1926
4	2568
5	3210
6	3852
7	4494
8	5136
9	5778

Now ſuppoſe it were required to divide 146728 by 642, I ſet down the Diviſor and Dividend as uſual, 642)146728, and conſider out of how many of the foremoſt Figures or Places of the Dividend I muſt firſt take the Diviſor. It is plain at firſt Sight that three will not do, therefore I take four, *viz.* 1467, putting a Dot under the 7, to ſhew that the Diviſor reaches ſo far into the Dividend. Then aſking how often 642 is contained in 1467, I look in the Table, but not finding that Number exactly, I look for the next leſs, which is 1284; and this ſtanding over againſt the Number 2, I thereby know that the Diviſor 642 is twice contained in the Dividual 1467. I therefore put down 2 in the Quotient, and ſubtracting 1284 from 1467 I find 183 remaining, which being ſet down the Work ſtands thus:

```
642)146728(2
        ·
     1284
     ————
      183
```

Proceeding as uſual, I now make a Dot under 2, the next Figure in the Dividend, and bring it down to the Remainder 183, which gives me 1832 for a new Dividual. Then enquiring how often 642 is contained in 1832, I find by the Table it is no more than twice, for 1284 is again the next leſs Number; I therefore put down 2 in the Quotient, and ſubtracting 1284 from the Dividual 1832, there remain 548, the Work appearing thus:

```
642)146728(22
      ..
    1284
    ----
     1832
     1284
    -----
      548
```

I then make a Dot under 8, the laſt Figure in the Dividend, and bring it down to the remaining 548, whereby I have another Dividual to work upon, *viz.* 5488, and therefore I enquire how often 642 can be had in that Number, which I find by the Table is 8 times; for 5136, the next leſs Number to 5488, ſtands oppoſite to 8, which being placed in the Quotient, and 5136 ſubtracted from 5488, the Remainder is 352; and there being no more Figures to bring down from the Dividend, the Work is finiſhed, and ſtands as follows:

642)

642)146728(228$\frac{352}{642}$

...

1284

1832

1284

5488

5136

352

This Method of making a Table of the Diviſor will be of great Service to young Scholars, till they are well practiſed in Diviſion; and by beginning with ſmall Sums, and ſo proceeding to greater, they will ſoon acquire a ſufficient Knowledge in this Branch of Arithmetic. I ſhall ſet down one Example more with the ſame Diviſor, without a verbal Explanation of the Work, that the Learner may try how he he can make uſe of the Table. Divide 2579683 by 642, and the Work when finiſhed will ſtand thus :

$$642)2579683(4018\tfrac{127}{642}$$

• • • •

2568

1168
642

5263
5136

127

Q. Is there not ſometimes a ſhort Way of working in *Diviſion*, as well as in *Multiplication?*

A. Yes ; I have already ſpoken of ſuch a Method when the Diviſor is only a ſingle Figure, and in ſome

Caſes the Work may be ſhortened, even when the Diviſor conſiſts of ſeveral Figures.

Firſt, When the Diviſor has one, two, or more Cyphers to the Right Hand, ſeparate them from the ſignificant Figures by a Comma; and in like Manner ſeparate the ſame Number of Figures or Cyphers from the Right Hand of the Dividend; then divide the remaining Figures of the Dividend towards the Left Hand by the ſignificant Figures of the Diviſor, and if there be any Remainder, bring down to it the Figures cut off from the Dividend, for they are always a Part of the Remainder; and ſometimes the Whole: As in the following Examples.

Example 1.

24,00)482,20($20\frac{220}{2400}$

48

---

220 Rem.

Example 2.

3,000)169,242($56\frac{1242}{3000}$

15

19

18

1242 Rem.

Secondly, It appears from the foregoing rule, that when we are to divide by 10, 100, 1000, *&c.* we have nothing more to do than to ſeparate ſo many Figures or Cyphers from the Right of the Dividend as there are Cyphers in the Diviſor; and the Figures on the Left Hand of the Mark of Separation are the *Quotient*, as thoſe on the Right are the *Remainder.*— Suppoſe, for Example, the Number 31467 were to be divided by 10, 100, and 1000, the Work and the Anſwer appear at one View as follows:

10)3146,7
100)314,67
1000)31,467

In the firſt Inſtance 3146 is the Quotient, and 7 the Remainder; in the ſecond the Quotient is 314, and the Remainder 67; and in the laſt the Quotient is 31, and 467 the Remainder.

Q. How do you divide Numbers of divers Denominations?

A. Suppoſe 65*l.* 17*s.* 6*d.* were to be divided amongſt 5 Men, ſet down the Sum as follows:

l. *s.* *d.*
5)65 : 17 : 6(

Here I conſider how many times 5 may be had in 6, the firſt Figure of the Pounds, which being once, I ſet down 1 in the Place of the Quotient, and carry the Overplus 1 to the 5, the next Figure in the Pounds, which makes it 15. Then I ſay, 5 in 15 may be had three times, and therefore I put down 3 in the Quotient. Having done with the Pounds I proceed to the Shillings, ſaying, the Fives in 17 are 3 and 2 over; I therefore ſet down 3 in the Quotient, and conſidering that the Overplus 2 are 2 Shillings, or 24 Pence, I add 24 to the 6 in the Place of Pence, which makes them 30 in all. Then I ſay, 5 out of 30 may be had 6 times, and nothing remains; therefore I put down 6 in the Quotient, and ſo the Work being finiſhed ſtands thus:

l. *s.* *d.* *l.* *s.* *d.*
5)65 : 17 : 6(13 : 3 : 6

For another Example, let it be required to divide 4*l.* 15*s.* 6*d.* among 6 Men, and the Work when compleated will appear as follows:

l. *s.* *d.* *l.* *s.* *d.*
6)4 : 15 : 6 (0 : 15 : 11

In this Inſtance, as I cannot take the Diviſor 6 from the 4 Pounds in the Dividend, I put down a Cypher in the Quotient, and conſidering how many Shillings are contained in 4 Pounds, *viz.* 80, I add 80 to the 15 in the Place of Shillings, which makes 95 in all. Then I enquire how many times 6 in 95, which being 15 times and 5 over, I ſet down 15 in the Quotient; and the Overplus 5 Shillings containing 60 Pence, I add 60 to the following 6, making 66 in all; in which Number the Diviſor being contained juſt 11 Times without any Remainder, I ſet down 11 in the Quotient, and the Operation is finiſhed, ſhewing 15*s.* 11*d.* to be each Man's Share of the Money.

Q. In what Manner is *Diviſion* to be proved?

A. Multiplication and Diviſion mutually prove each other: For as in Multiplication, if you divide the Product by the Multiplier, the Quotient will be the Multiplicand; ſo in Diviſion, if you multiply the Quotient by the Diviſor, (taking the Remainder if there be any) the Product will be the Dividend, or elſe the Work is not rightly performed.

To make this plain by an Example, let us divide 8280 by 24, and the Work will appear as follows:

```
24)8280(345
   72
   ---
   108
    96
   ---
   120
   120
   ---
     0
   ---
```

In order to prove this according to the Rule laid down, we muſt multiply the Quotient 345 by the Diviſor 24, and the Product will be the Dividend 8280, as appears below:

```
 345
  24
----
1380
690
----
8280
----
```

N. B. Diviſion may alſo be proved by Diviſion; that is, if you divide the Dividend by the Quotient, the Quotient will be your former Diviſor: As in the Example above, if 345 were made the Diviſor, the Quotient would be 24.

CHAP. VI.

Of REDUCTION.

Q. WHAT other Rules are there in *Arithmetic?*

A. We have now gone through the five *fundamental* Rules, upon which all the ſucceeding ones depend, and are come to that called REDUCTION, which is uſually taught immediately after *Diviſion*.

Q. What is *Reduction?*

A. It is an Application of Multiplication and Diviſion, ſhewing how to reduce Numbers of one Denomination to another, ſtill retaining the ſame Value, tho' in different Terms. *Firſt*, All great Names are brought into ſmall by multiplying by ſo many of the little ones as make one of the great. Thus any Number of Pounds multiplied by 20 are reduced into Shillings, becauſe 20 Shillings make a Pound: Shillings multiplied by 12 are reduced into Pence, and theſe multiplied by 4 are reduced into Farthings, as has been obſerved already in the Chapter of *Multiplication*. —*Secondly*, Small Names are brought into greater by Diviſion; as Farthings into Pence by being divided by 4, Pence into Shillings by 12, and Shillings into Pounds by 20: For Multiplication and Diviſion are two contrary Operations.—*Thirdly*, In changing one Sort of Money, Weight, Meaſure, &c. into another, both muſt be reduced into the ſame Denomination, and the one divided by the other.

Q. Give me an Example of the firſt Sort of *Reduction.*

A. Let us begin with Money, and aſk, *How many Shillings, Pence, and Farthings there are in* 27 *Pounds.* To know this you are to multiply the Pounds by 20, the Shillings by 12, and the Pence by 4, and the Work will ſtand as follows:

27 *l.*
20

540 Shillings.
12

1080
540

6480 Pence.
4

25920 Farthings.

N. B. Pounds may be brought into Pence at once by multiplying by 240, or into Farthings by multiplying by 960, there being ſo many Pence and Farthings in a Pound.

One Example more of this Kind will be ſufficient; as, *How many Farthings are there in* 12l. 7s. 6d. ?—— The Work when finiſh'd appears thus:

l. *s.* *d.*
12 : 7 : 6
20

247 Shillings.
12

500
247

2970 Pence.
4

11880 Farthings.

Here the Manner of working is the ſame as in the firſt Example, only in multiplying the Pounds we take in the 7 Shillings, to which Name the Pounds are reduced by that Multiplication; and in multiplying the Shillings we take in the 6 Pence, that being the Name to which the Shillings are reduced.

A few more Examples in *Weights* and *Meaſures* will make this Rule very eaſy and intelligible to the young Scholar.

In 21 *C. wt. how many Quarters and Pounds?*—Here you are to conſider, that as 4 Quarters make a Hundred Weight, multiplying the Hundreds by 4 will reduce them to Quarters; and as 28 Pounds make a Quarter, multiplying them by 28 will bring them into Pounds, and give an Anſwer to the Queſtion, as follows:

```
  21 C. wt.
   4
  ——
  84 Quarters.
  28
 ———
 672
168
————
2352 Pounds.
————
```

If the Pounds were to be multiplied by 16, the Product would be the Number of Ounces; and if the Ounces were alſo multiplied by 16, the Product would give the Number of Drachms contained in 21 Hundred Weight.

In 123 *Hogſheads how many Gallons and Pints?*—Here you are to remember (or conſult your Tables) that 63 Gallons make a Hogſhead, by which

Number you are therefore to multiply to reduce the Hogſheads to Gallons; then multiply the Gallons by 8 to reduce them to Pints, and the Work will appear as follows:

123 Hogſheads.
63

369
738

7749 Gallons.
8

61992 Pints.

In 546 *Yards how many Feet and Inches?*—Multiply the Yards by 3 to bring them into Feet, and the Feet by 12 to bring them into Inches; and the Work will ſtand thus:

546 Yards.

$$
\begin{array}{rl}
546 & \text{Yards,} \\
3 & \\
\hline
1638 & \text{Feet.} \\
12 & \\
\hline
3276 & \\
1638 & \\
\hline
19656 & \text{Inches:} \\
\hline
\end{array}
$$

In 12 *Weeks how many Days, Hours, and Minutes?*—Multiply the Weeks by 7 to reduce them to Days, the Days by 24 to reduce them to Hours, and the Hours by 60 to bring them into Minutes.

12 Weeks.
7
84 Days.
24
336
168
2016 Hours.
60
120960 Minutes.

Theſe Examples may ſuffice for this Kind of Reduction, which is called Reduction *Deſcending*, becauſe it brings great Denominations into ſmaller. The next Branch of this Rule, which is performed by Diviſion, is called Reduction *Aſcending*, becauſe it brings leſſer Denomination into greater.

Q. What Examples have you of this Sort of Reduction?

A. The foregoing Queſtions reverſed will be ſo many Examples in this Branch of Reduction, and each Operation will prove the Truth of the other. Firſt then,

How many Pence, Shillings, and Pounds in 25920 *Farthings?* In this Caſe, as I before multiplied by 20, 12, and 4, I muſt now divide by 4, 12, and 20.

4) 25920 Farthings.

12) 6480 Pence.

20) 540 Shillings.

27 Pounds.

Again: *How many Pounds in* 11880 *Farthings?*—Here, dividing as in the laſt Example, the whole Work will appear as follows:

4)

4) 11880 Farthings.

12) 2970 Pence.

20) 247 Shillings and 6 Pence.

12 Pounds and 7 Shillings.

Thus I find that 11880 Farthings are equal to 12*l.* 7*s.* 6*d.* And it would have amounted to the ſame thing if I had divided 11880 by 960, the Number of Farthings in a Pound. Here it may be obſerved, that whatever odd Money, Weight, or Meaſure we take in as we multiply the ſeveral Denominations in *Reduction Deſcending*, the ſame will be a Remainder in *Reduction Aſcending*; as the 7*s.* 6*d.* in the above Example.

In the next Place, *How many Quarters and Hundred Weight are there in* 2352 *Pounds?*—This is anſwered by dividing firſt by 28 the Number of Pounds in a Quarter, and then by the Number of Quarters in a Hundred.

Pounds.
28)2352(84 Quarters.
224
——
112
112
——
0
——

Here the Quotient ſhews the Number of Quarters to be 84, and this divided by 4 ſhews the Number of Hundreds, namely 21.—Thus theſe Examples prove the Truth of the correſponding ones in the former Part of *Reduction*, as you will ſee by comparing them together.

I apprehend it is needleſs to run through more Examples of this Kind, thoſe already given being ſufficient to ſhew the Nature of Reduction, *Aſcending* and *Deſcending*, ſo far at leaſt as they are ſeparately employed: But it is proper by a few Examples, to explain the *Third Part* of this Rule, (if I may call it ſo) wherein *Multiplication* and *Diviſion* are both concerned in anſwering the ſame Queſtion.

Q. What is the Buſineſs of this third Part of Reduction?

A. It is uſed in changing one Sort of Money, Weight, Meaſure, *&c.* into another; as Foreign Money into Sterling, or Sterling into Foreign — For Example, *In* 459 *French Crowns, at* 4*s.* 6*d. each, how many Pounds Sterling?*

This, and other Queſtions of the ſame Nature may be anſwered ſeveral Ways as the Scholar himſelf will find nd by a little Conſideration and Prac-ice: But the moſt obvious Method f proceeding in this Caſe is to mul-iply the Crowns by 9 to bring them nto Sixpences, and to divide the Product by 40 to bring them into Pounds; as each Crown contains 9 Sixpences, and each Pound 40.—See the Work as follows:

459 *French* Crowns.
9

4,0)413,1(11 Sixpences remaining.

103 Pounds.

So that the Anſwer is 103*l.* 5*s.* 6*d.* —The Reverſe of this, namely, *How many French Crowns there are in* 103*l.* 5*s.* 6*d.* Sterling is found by multiplying 103 by 40, taking in the 11 Sixpences, and then dividing the Product by 9; as is eaſy to be underſtood without further Explanation.

In 524 *Moidores, at* 27 *Shillings eac* *how many Guineas?* Here multipl the Moidores by 27 to reduce the to Shillings, and then divide the Product by 21, the Quotient will give th Anſwer required.

524 Moidores.
27
———
3668
1048
———
21)14148(673 Guineas.
126
———
154
147
———
78
63
——
15 Shillings remaining.
——

Thi

This Work may be performed in a orter Manner, for when the Moi- ores are reduced to Shillings, you ay divide them by 3, and then that uotient by 7, and the laſt Quotient ill be the Anſwer to the Queſtion, appears below:

)14148 Shillings in 524 Moidores.

7)4716

673 Guineas and 5 remaining; that is, five 3's or 15 Shillings.

To underſtand the Reaſon of this, ou muſt conſider, that the Shillings eing divided by 3 are thereby re- uced to ſo many 3 Shillings as the uotient expreſſes, and conſequently heſe being divided by 7 are reduced Guineas; becauſe 7 times 3, or 3 imes 7, make 21, the Number of hillings in a Guinea. Obſerve alſo, that the 5 remaining are ſo man times 3 Shillings, the Remainder b ing always of the ſame Name wi the Dividend.

But there is ſtill a more compendious Method of performing th Work, which is worth the Scholar Notice, and the beſt that can be practiſed whenever the Caſe will admit o ſuch Contraction. As 9 is one Thir of 27, the Shillings in a Moidor multiply the given Number of Moidores by 9, and the Product ſhew how many 3 Shillings they contai which being divided by 7 are reduce to Guineas, as above; and ſo th whole Work appears in the following ſmall Compaſs:

524 Moi

524 Moidores.
9

7)4716 Three Shillings.

673 Guineas, 15 *s.*

In 215 Portugal *Pieces, of* 36 *Shillings each, how many Guineas?* This Queſtion may be anſwered in the ſame ſhort Manner as the former, for as 3 times 12 is 36, and 3 times 7 is 21, you need only multiply 215 by 12, and divide the Product by 7, and the Quotient is the Anſwer required.

215 Pieces.
12

7)2580 Three Shillings.

368 Guineas, 12 *s.*

In 246 Venetian *Ducat, at* 4*s*. 4*d*. *each, how many Pounds Sterling* ?— To anſwer this, multiply the give Number of Ducats by 52, to brin them into Pence; then divide th Pence by 12 to bring them into Shillings, and theſe by 20 to bring the into Pounds; and the Work will appear as follows:

```
       246 Ducats.
        52
      ----
       492
      1230
      ----
   12)12792 Pence.
      -----
   2,0)106,6 Shillings.
        ----
        53 Pounds, 6 Shillings.
        ----
```

In 672 *Ells* Flemiſh, *how many El* Engliſh ?—Here remember (as you Table

Tables inform you) that the *Flemiſh* Ell is 3 Quarters of a Yard, and the *Engliſh* Ell 5 Quarters; therefore multiply the given Number of *Flemiſh* Ells by 3 to reduce them to Quarters, then divide the Quarters by 5, and the Quotient will be the Anſwer to the Queſtion, as appears from the Work at length:

```
  672 Ells Flemiſh.
    3
  ———
5)2016 Quarters.
  ———
  403 Ells Engliſh, 1 Quarter.
  ———
```

I forbear to give the Reverſe of theſe Queſtions, as thinking the Manner of ſtating and working them will not be difficult to the young Scholar, but a proper Exercife for the Trial of his Ingenuity.

N. B. There is a compendious Way of reducing Ells *Flemiſh* to Ells *Engliſh*, namely, to multiply them by 6, and cut off the Right-hand Figure of the Product, in the following Manner:

672 Ells *Flemiſh*.
6

Ells *Engliſh* 403|2

In 456 *Ells* Engliſh, *how many Yards?*—To anſwer this Queſtion multiply by 5 to bring them into Quarters, and divide by 4 to reduce them to Yards, as follows:

456 Ells *Engliſh*.
5

4)2280 Quarters.

570 Yards.

N. B. Yards are compendiouſly reduced to Ells by multiplying by 8, and cutting off the Right-hand Figure or Cypher of the Product, thus:

```
      570 Yards.
        8
     ----
Ells 456|0
     ----
```

Enough has been ſaid to explain the Nature of *Reduction*; I ſhall therefore conclude this Rule with one Queſtion and its Anſwer, leaving the Operation as a Trial for the Learner.

In 722 French *Livres*, at 20*d. each, how many Pieces of Eight at* 4*s.* 4*d.* —Anſwer, 277 Pieces and 3 Shillings over.

CHAP. VII.

Of the Rule *of* PROPORTION, *or the* Rule of THREE, *Direct, Indirect, and Double.*

Q. WHAT is the Uſe of this Rule?

A. It is of very great Uſe both in the Buſineſs of Common Life and in the Sciences, and is therefore, by way of Eminence, frequently called the *Golden Rule.*

Q. What is the Reaſon of its other Names?

A. It is called the *Rule of Three,* or *Rule of Proportion,* becauſe by three Numbers given we find out a fourth, which bears ſuch Proportion to the third, as the ſecond bears to the firſt.

Q. How are Queſtions to be ſtated for working according to this Rule?

A. With reſpect to the ſtating any

Queſtion obſerve, that of the three given Numbers two always contain a Suppoſition, and the third a Demand. The Number then on which the Demand lies, that is, to which the Interrogatives *What?* *How many?* &c. are annexed, muſt always be ſet in the *third* Place, namely, on the Right Hand; and of the other two Numbers, that which is of the ſame Name or Kind with the Right-hand Number muſt be ſet on the Left-Hand, or in the *firſt* Place; ſo that the remaining Number muſt of conſequence ſtand in the *ſecond*, that is between them. Suppoſe, for Inſtance, it were aſked, *If* 3 *Oranges coſt* 6 *Pence, what will* 24 *Oranges coſt?* the Numbers for working muſt be ſtated thus:

Or. *Pence.* *Or.*
If 3 — — 6 —— 24

Here you ſee that 24, the Number

of which the Queſtion is aſked, is placed on the Right Hand; the Number 3, which is of the ſame Kind, as it likewiſe denotes *Oranges*, is placed on the Left, and the Number 6, which is of a different Kind, as ſignifying *Pence*, is ſet in the Middle, according to the above Directions.

Q. How is the Work to be performed?

A. Multiply the ſecond and third Numbers together, then divide the Product by the firſt, and the Quotient will give the Anſwer to the Queſtion, in the ſame Name with the middle Number, that is, in *Pence*; for you are to obſerve, that whatever Name the ſecond Number bears at the Time of multiplying, the Quotient alſo bears when the Diviſion is performed.—See the Work at length.

Or. Pence. Or.
If 3 — 6 — 24
6

3) 144 (

Anſw. 48 Pence, or 4s.

This Queſtion ſtated the contrary Way will ſerve for another Example, and prove the Truth of the former Operation.—Say then, If 24 *Oranges coſt* 48 *Pence, what is the Price of* 3 *Oranges at the ſame Rate* ?—In this Caſe the Numbers and Work according to the foregoing Directions will appear as follows :

If 24 — 84 — 3
3
24)144(6 *Anſwer.*
144
0

Here the firſt and ſecond Numbers multiplied together produce 144, which being divided by 24, the firſt Number, the Quotient is 6, that is, 6 Pence, the Price of 3 Oranges.— But for the better underſtanding this Rule, it is neceſſary to give ſome more Examples.

If 6 *Gallons of Brandy coſt* 2l. 5s. *what will* 134 *Gallons coſt?*—This is to be ſtated and worked as follows:

```
Gall.      l.    s.     Gall.
If 6 —— 2 — 5 —— 134
          20             45
          —             ——
          45            670
                       536
                       ——
                     6)6030
                       ——
                   2,0)100,5
                      ———
              Anſw. 50l. 5s.
                     ———
```

Here you ſee the ſecond or middle Number being of different Denominations, that is *Pounds* and *Shillings*, is reduced to the loweſt of thoſe Names, (*viz.* Shillings) by Multiplication, and this muſt always be done when it conſiſts of ſeveral Names; for if it be of Pounds, Shillings, and Pence, it muſt be reduced to Pence, that is, into the loweſt Denomination mentioned.—The Middle Number in this Example being thus reduced, the three Numbers for the Operation are 6 — 45 — 134: I therefore multiply the third Number by the ſecond, that is, 134 by 45, according to the Directions given, and the Product is 6030; which being divided by 6, the firſt Number, quotes 1005 Shillings, the Name of the middle Number 45; and theſe divided by 20 give 50*l.* 5*s.* for the Anſwer to the Queſtion.

If 4s. 6d. *purchaſe* 1 *Yard and* ½ *of Linen, how many Yards will* 12l. 12s. *purchaſe*?

In this Queſtion you are to obſerve, that the firſt and third Numbers are of different Denominations, the one being Shillings and Pence, and the other Pounds and Shillings. Now the firſt Buſineſs is to reduce theſe to the ſame Name, which muſt be the loweſt mentioned; and this muſt always be done in the like Caſes, immediately after the ſtating of the Queſtion. Here then we are to bring the firſt and third Numbers into *Sixpences*; and the Middle Number being alſo of different Names, that is, Yards and Half Yards, muſt be brought into the latter, agreeable to the Directions already given. When this is done, we are to multiply and divide according to the Rule, and the Work will ſtand as follows:

s. *d.* *Yards.* *l.* *s.*
If 4 : 6 —— $1\frac{1}{2}$ —— 12 : 12
2 2 40
—— —— ——
9 3 504
3
——
9)1512
——
2) 168 Half Yards.
——
Anſw. 84 Yards.

Here you ſee, the firſt Number is multiplied by 2 to reduce it to Six-pences, becauſe 2 Six-pences make 1 Shilling; the ſecond is alſo multiplied by 2 to reduce it to Half Yards, becauſe 2 Half-yards make a Yard; and the third is brought into Sixpences by multiplying it by 40, becauſe 40 Sixpences make one Pound. This being done, the Numbers for working are 9—3—504; the third therefore being multiplied by the ſecond, the Product is 1512, and this being divided by the firſt Number, the Quotient is 168, which are Half Yards, the Name to which the middle Number was reduced; and theſe being divided by 2 the Quotient is 84 Yards, which is the Anſwer to the Queſtion.

N. B. If there be any Remainder after you have multiplied your ſecond and third Numbers together and divided by the firſt, ſuch Remainder is

Part of an Unit of the Quotient, the Value of which is thus found. Multiply it by that Number of the next inferior Denomination which makes one of the Quotient, and divide the Product by the ſame Diviſor you made uſe of in your firſt Diviſion: Suppoſe, for Inſtance, the Quotient be Pounds, multiply the Remainder by 20, and divide the Product by the firſt Diviſor, and the Quotient will be Shillings; then if any Thing remains, multiply it by 12, divide by the ſame Diviſor, and the Quotient will be Pence; and if there be ſtill a Remainder, multiply by 4, divide as before, and the Quotient will be Farthings.

Note alſo, When the firſt of the three given Numbers is an Unit, or One, you cannot divide by it according to the Rule, for 1 neither divides nor multiplies. In this Caſe, when you have multiplied the ſecond and third Numbers together, the Product is alſo the Quotient, giving the Anſwer to the Queſtion in the Denomination of the middle Number, as appears in the following Example.

If 1 *Pound of Silk coſt* 17s. *what will be the Price of* 264 *Pounds at the ſame Rate* ?

lb. *s.* *lb.*
If 1 — 17 — 264
17
1848
264
Anſwer 4488 Shillings.

Here the Queſtion is anſwered by Multiplication only, but the Anſwer being in Shillings is not readily underſtood till it be reduced to Pounds by dividing by 20, and then it appears to be 224*l.* 8*s.*

These Examples may ſuffice to ſhew the Manner of working all Queſtions in the *Direct* Rule of Three, that is, vhen the ſecond and third Numbers are multiplied together, and the Product divided by the firſt: I ſhall now give a few Examples in the Rule of Three *Indirect* or *Inverſe*, wherein he firſt and ſecond Numbers are nultiplied together, and their Product divided by the third.

Q. How are Queſtions ſtated in his Rule?

A. The ſame Way as in the *Direct* Rule; for the firſt and third Numbers muſt be of one Name, or ſo reduced; the Number of which the Queſtion is aſked muſt ſtand in the hird Place; and the Number of a diferent Denomination in the Middle.

Q. How is the Anſwer to be found?

A. The Quotient, as before, will e the Anſwer, and of the ſame Name with the middle Number. For Example: If *nine Men would build a Wall in* 15 *Days, how many Men muſt be employed to build it in* 5 *Days?*

In ſtating theſe Numbers obſerve the above Rule, and the Work will appear as follows:

Days. *Men.* Days.
If 15 — 9 — 5
9

5) 135

Anſw. 27 Men.

Here you ſee, the firſt Number being multiplied by the ſecond, and the Product divided by the third, the Quotient is 27, the Anſwer to the Queſtion. And you may alſo obſerve the *indirect Proportion* the Numbers bear to each other; for ſo much as the third is leſs than the firſt, ſo much is the fourth greater than the ſecond, and on the contrary the fourth is always ſo much leſs than the ſecond, as the third is greater than the firſt.— But let us have another Example.

If 60 *Yards of Serge, of* 3 qrs. *wide, will line a certain Number of Coats, how many Yards will line the ſame Number when the Serge is but Half Yard wide?*

Here remember that your firſt and third Numbers muſt be of one Name; therefore call the Half Yard *2 qrs.* and ſtate the Queſtion as follows.

qrs. *yards.* *qrs.*
If 3 — 60 — 2
3

2)180

Anſ. 90 Yards.

Again: If *in* 12 *Months* 100 l.

Principal gain five Pounds Intereſt, what Principal will gain the ſame Intereſt in 5 Months?

M. *l.* *M.*
If 12 — 100 — 5
12

5)1200

Anſw. 240 l.

Once more: *If 6 Men dig a Trench in 24 Days, in what Time would 18 Men do the ſame Work?*

Men. *Days.* *Men.*
If 6 — 24 — 18
6

18)144

Anſw. 8 Days.

Q. How are we to know whether a Queſtion belongs to the *Direct* or to the *Indirect* Rule of Three?

A. If the third Number is more than the firſt, and requires more than the ſecond to anſwer the Queſtion; or if it is leſs than the firſt, and requires leſs than the ſecond, then it is *Direct*: But if the third Number is more than the firſt, and requires leſs than the ſecond, or being leſs than the firſt requires more than the ſecond, then it is *Indirect*.—After all there is no Neceſſity for puzzling the Learner with this Diſtinction; for as the Method of ſtating a Queſtion is the ſame whether the Proportion be *Direct* or *Indirect*, ſo the following Rule for the Operation will be ſufficient in all Caſes, *viz.* Conſider whether the Anſwer to the Queſtion will be more or leſs than the ſecond Number; if *more*, the *leſſer* of the two Extremes, (that is, of the firſt and third Numbers) muſt be the Diviſor; if *leſs* the *greater.*

Q. Which way do you prove your Work in the *Rule of Three?*

A. It may be proved by a contrary Stating and Operation, as has been already intimated; but the eaſieſt Method, in Queſtions that are *Direct*, is to multiply the firſt and fourth Numbers together, and the ſecond and third, and if the two Products are alike the Work is right, In Queſtions that are *Indirect*, if the Product of the third and fourth multiplied together agree with that of the firſt and ſecond, it proves the Truth of the Operation.

Q. What is meant by the *Double Rule of Three?*

A. It is ſo called becauſe Queſtions in it are generally anſwered by *two* Statings, there being five Numbers given to find out a ſixth. For Example:

If 100*l. Principal gain* 5*l. Intereſt in* 12 *Months, what will* 246*l. gain in* 7 *Months* ?

Here the Buſineſs is to find out what Intereſt 246*l.* will gain in 12 Months at the given Rate, and from thence what they will gain in 7 Months. The firſt Stating therefore, and Operation, according to the Rules already preſcribed, will be as follows:

Pr. *Int.* *Pr.*
If 100 — 5 — 246
5
1,00)12,30
20
1,00)6,00 *Anſw.* 12*l.* 6*s.*

Obſerve here, that the Product of 246, multiplied by 5, is 1230, which being divided by 100 the Quotient is 12, *i. e.* ſo many Pounds, and the

Remainder is 30. Now the Value of this Remainder is found by the Rule I have already given for that Purpoſe, namely, multiplying it by 20, and dividing the Product by 100, the firſt Diviſor. This done, the Quotient is 6, which are Shillings; and ſo 12*l.* 6*s.* is the Anſwer to this Stating of the Queſtion.

As to the ſecond Stating, take Notice, that the Anſwer to the firſt Stating is always the Middle Number of the laſt; and therefore 12*l.* 6*s.* being placed in the Middle, and the given Months on the Right and Left according to the Rule they will ſtand thus:

Mo. *l.* *s.* *Mo.*
If 12 — 12 : 6 — 7
20
———
246
7
———
12)1722
———
2,0)14,3 - 6*d.*
———
Anſw. 7*l.* 3*s.* 6*d.*

Here the Middle Number is multiplied by 20 to bring it into Shillings and theſe being multiplied by 7, the leſſer Extreme, and divided by 12 the greater, are reduced to Pounds by dividing by 20, the Quotient being 7*l.* 3*s.* 6*d.* which is the Intereſt that 246*l.* will gain in 7 Months at the Rate of 5 *per Cent. per Annum.*

N. B. The foregoing Queſtion, and others of the ſame Nature, may be anſwered at one Stating of the 5 Numbers according to the following Directions. Let the three Numbers that contain the Suppoſition ſtand firſt, and after them the two that contain the Demand: And this muſt be done in ſuch Manner that the firſt be of the ſame Denomination with the fourth, the ſecond of the ſame with the fifth, and the third with the Anſwer requir'd. When the Numbers are thus ſtated, multiply the two firſt together for a Diviſor, and the three laſt for a Dividend, and the Quotient will be the Anſwer in the ſame Name with the middle Number. I ſhall put down the State of the former Queſtion, and the Beginning of the Work, the finiſhing of which I leave for the Exerciſe of the Learner.

If

	Pr.		*M.*		*Int.*		*Pr.*		*M.*
If	100	—	12	—	5	—	246	—	7
	12						5		
	1200						1230		
							7		
							1200)8610		

Here the two firſt Numbers multiplied together produce 1200 for a Diviſor, and the three laſt 8610 for a Dividend, which being rightly divided, and the Remainders that will come out in the Operation being alſo multiplied and divided according to Rule, the Anſwer to the Queſtion will be found to be the ſame as above, *viz.* 7*l.* 3*s.* 6*d.*

In Queſtions where the Propoſition is *Indirect*, place your Numbers ſo as that your ſecond and fourth may be of one Denomination, and your third and fifth. Then multiply your firſt, ſecond, and fifth Numbers together for a Dividend, and your third and fourth for a Diviſor.

For Example: *If* 2 *Men reap* 12 *Acres in* 6 *Days, how many Men will reap* 192 *Acres in* 24 *Days* ?—The ſtating and working of this Queſtion is as follows:

	Men.		*Days.*		*Acr.*		*Days.*		*Acr.*
If	2	—	6	—	12	—	24	—	192
			2				12		12
			12				288	)	2304(8
									2304
									0

Here according to the Rule, the firſt, ſecond, and fifth Numbers multiplied into each other give 2304 for a Dividend; which being divided by

288

288, the Product of the third and fourth Numbers quotes 8, the Anſwer to the Queſtion.

Having ſufficiently explained the *Rule of Three*, I now proceed to the *Rules of Practice*.

CHAP. VIII.

Of PRACTICE; *and of* TARE *and* TRETT.

Q. WHAT do you mean by *Rules of Practice*?

A. In effect they are only certain compendious Methods of working the *Rule of Three*; but being of excellent Service for the quick Diſpatch of Buſineſs, and generally made uſe of by Merchants and Tradeſmen in caſting up the Price of their Commodities, they are not improperly called *Rules of Practice*.

Q. In what Caſes are they chiefly uſed?

A. In ſuch Queſtions of the *Rule of Three* as have an *Unit* for their firſt Number; that is, when the Price of *one* Thing is given to find the Price of *many:* And to work Queſtions by theſe Rules it is neceſſary to have in Memory the *Aliquot Parts* of a Pound and Shilling.

Q. What is meant by *Aliquot Parts*?

A. No more than *Even Parts*, or ſuch Parts of any Number or Quantity as being taken certain Times are exactly equal to the Whole. Thus 3 is an *Aliquot* or *Even* Part of 12, becauſe being taken four times it is juſt equal to 12; *i. e.* it is one *Fourth* of that Number.

Q. Which are the even Parts of a Shilling?

A. Learn them from the following Table.

d. 1

d.			*Of a Shilling.*
1	is $\frac{1}{12}$	that is,	one Twelfth.
$1\frac{1}{2}$	- $\frac{1}{8}$		one Eighth.
2	— $\frac{1}{6}$		one Sixth.
3	— $\frac{1}{4}$		one Fourth.
4	— $\frac{1}{3}$		one Third.
6	— $\frac{1}{2}$		one Half.

Q. What is the Uſe of this Table?

A. When any of theſe even Parts of a Shilling is the Price of one Yard, Ell, Pound, *&c.* divide the given Number (of which you would know the Price) by ſuch Part, and the Quotient will be the Anſwer to the Queſtion in Shillings.—An Example or two will make this plain.

Suppoſe you would know the Price of 672 *Oranges, at the Rate of* 1 *Penny an Orange.* You ſee by the Table that a Penny is one 12th Part of a Shilling, therefore divide 672 by 12, and the Quotient is the Anſwer:

12)672

Anſwer 56 Shillings.

Again: *What would the ſame Number coſt at three Half-pence an Orange?* —The Table ſhews you that 1*d*.$\frac{1}{2}$ is one 8th Part of a Shilling, therefore divide 672 by 8, and you will have the Anſwer:

8) 672

Anſwer 8|4 Shillings.
or 4*l*. 4*s*.

N. B. By cutting off the laſt Figure of any Number of Shillings with a Daſh, and halving the Figures on the Left, (which is the ſame thing as dividing by 20) you have the Number of Pounds; and the Figure on the Right Hand of the Daſh is Shillings, as you ſee in the Example above. And if when you have halved the

Left-hand Figures there be a Remainder, it is 10 Shillings, and as ſuch muſt be reckoned with the ſeparated Figure, as in the next Example.

At 2 Pence the Yard, what will 672 *Yards coſt?*—Two-pence, as you ſee in the Table, is one 6th Part of a Shilling, therefore divide by 6, and the Quotient gives the Number of Shillings.

6)672

Anſwer 11|2 Shillings.

---

or 5*l.* 12*s.*

Here, I ſay, the Half of 11 is 5 and 1 over, which being reckoned with the Figure on the Right-hand of the Daſh makes 12, and ſo the Anſwer to the Queſtion is 5 Pounds 12 Shillings.

After this Method may any Queſtion be reſolved where the Price is an even Part of a Shilling; but if the Price be not an even Part, as 5*d.* or any Number of Pence between 6 and 12, it will require more than one Diviſion, as will be ſeen in the following Examples.

At 5*d. an Ell, what will* 672 *Ells coſt*?—Here conſider that Five-pence contains two even Parts of a Shilling, *viz.* 3 Pence and 2 Pence; 3 Pence being $\frac{1}{4}$, and 2 Pence $\frac{1}{6}$ Part: Therefore divide 672 by 4 and by 6, and add the two Quotients together, the Total will be the Anſwer.

	672
d.	
3$\frac{1}{4}$	168
2$\frac{1}{6}$	112
Anſwer	28|0 Shillings.
or	14*l.*

N. B. It would have been the ſame Thing to have divided by 3 and 12, becauſe 4 Pence and 1 Penny (which are equal to 5 Pence) are $\frac{1}{3}$ and $\frac{1}{12}$ Parts of a Shilling.—Or if you firſt divide by 3 for the Groats, and then the Quotient by 4, becauſe 1 Penny is $\frac{1}{4}$ of a Groat, the Anſwer will be the ſame; as appears by the Work:

	672
d.	
$4\frac{1}{3}$	224
$1\frac{1}{4}$	56
	280

Again: *At 9 Pence a Peck, what will* 144 *Pecks coſt?*—Here 6 and 3, which are equal to 9 Pence, are the even Parts of a Shilling; therefore for 6 Pence take $\frac{1}{2}$ the given Number, and for 3 Pence $\frac{1}{4}$, and the Work will ſtand as follows:

144
d. ——
6½ 72
3¼ 36
——
Anſwer 10|8 Shillings.
——
or 5*l.* 8*s.*
——

If the Price be 10 Pence, you may take Half the given Number for 6*d.* and divide by 3 for the 4*d.* becauſe 6*d.* is Half, and 4*d.* one Third of a Shilling.—Or you may annex a Cypher to the Right of the given Number, (which is the ſame as multiplying by 10) and it hereby becomes Pence; which are brought into Shillings and Pounds by dividing by 12 and 20. For Example:

What will 126*lb. of Hops coſt, at* 10*d. per Pound?*—Annex a Cypher to 126, 126, and the Work will appear as follows:

12)1260

10|5

Anſwer. 5*l.* 5*s.*

At 11d. 1q. *a Yard what will* 752 *Yards coſt?*—Here you begin by taking one Half of the given Number for Sixpence, then Half of that for Threepence, Half of that again for Three Half-pence, and Half of that for Three Farthings, which makes juſt 11*d.* 1*q.* And theſe Sums added together, and divided as before, give the Anſwer to the Queſtion, as you will ſee by the Work at length.

752
6*d.* $\frac{1}{2}$ ———
376
188
94
47
———
70|5
———
Anſwer 35*l.* 5*s.*
———

Take one Example more, wherein the Price is Pence and Farthings.— *At* 3d. $\frac{3}{4}$ *an Ounce, what will* 324 *Ounces coſt?*

324
3*d.* $\frac{1}{4}$ ———
$\frac{3}{4}$—$\frac{1}{4}$ 81
20—3
———
10|1
———
Anſwer 5*l.* 1*s.* 3*d.*

Here, becauſe 3*d.* is $\frac{1}{4}$ of a Shilling, I divide the given Number by 4, and the Quotient is 81; which I alſo divide by 4, becauſe three Farthings is $\frac{1}{4}$ of Threepence, and the Quotient is 20, and 1 over. This Remainder I conſider as one Threepence, and therefore I ſet down 3; and adding the two Quotients together the Total is 101, which being divided by 20 gives the Anſwer as above.

I ſhall now give an Example or two wherein the Price is a Shilling, and ſame Number of Pence and Farthings: In which Caſe the Rule is, to let the top Line (*i.e.* the given Number) ſtand for the Shilling, and take the Parts, as before, for the remaining Pence.

At 13d. $\frac{1}{2}$ *a Yard, what will* 144 *Yards coſt?*—Here it is plain they will coſt 144 Shillings, and as many Pence and Half-pence; wherefore the given Number is reckoned as Shillings,

and

and $\frac{1}{8}$ of it taken for the 1*d.* $\frac{1}{2}$, which added together give 162 Shillings, equal to 8*l.* 2*s.* the Anſwer.

144
18

16|2

Anſwer 8*l.* 2*s.*

At 16d.$\frac{1}{2}$ *a Yard, what will* 143 *Yards coſt* ?—See the Work at length.

143
35—9
17—10$\frac{1}{2}$

19|6—7$\frac{1}{2}$

Anſwer 9*l.* 16*s.* 7*d.*$\frac{1}{2}$

Here the given Number ſtands for the Shillings as before, the Reaſon of which is evident; then I divide it by 4 for Threepence and the Quotient is 35 and 3 over, which being ſo many Threepences I ſet down 9: Then, for the remaining 1*d.*$\frac{1}{2}$ of the given Price, I take half that Quotient, which is 17 and 1 over; and this 1 being reckoned as a Shilling, and added to the 9 Pence, makes 21 Pence, the half of which is 10*d.*$\frac{1}{2}$. Theſe Sums being added together, and divided, give the Anſwer as expreſſed in the Work.

I now proceed to Queſtions wherein the *Aliquot* or *Even* Parts of a Pound are to be conſidered, which you may learn from the following Table:

s.	d.			Of a Pound.
1	0	is $\frac{1}{20}$	that is,	one 20th
1	8	— $\frac{1}{12}$		one 12th
2	0	— $\frac{1}{10}$		one 10th
2	6	— $\frac{1}{8}$		one 8th
3	4	— $\frac{1}{6}$		one 6th
4	0	— $\frac{1}{5}$		one 5th
5	0	— $\frac{1}{4}$		one 4th
6	8	— $\frac{1}{3}$		one 3d
10	0	— $\frac{1}{2}$		one half

In Queſtions where the Price is 1*s.* only, you have nothing to do but to divide by 20, which you know is done by cutting off the Right-hand Figure of the given Number for Shillings, and taking Half of the Left-hand Figures for Pounds. In like Manner, If the Price be 1*s.* 8*d.* divide by 12, if it be 2*s.* divide by 10; if it be 2*s.* 6*d.* divide by 8; and ſo of any other even Part of a Pound; which being learnt from the Table, and the Operation being ſo eaſy, Examples of this Kind would be ſuperfluous.

If the Price does not conſiſt of even Parts, divide it into ſuch, and the Sum of them will be the Anſwer to the Queſtion. For Example: *At 3 Shillings an Ell, what will 288 Ells coſt?*

288

$\frac{1}{10}$	for 2*s.*	28*l.*	16*s.*
$\frac{1}{20}$	for 1*s.*	14*l.*	8*s.*
	Anſwer	43*l.*	4*s.*

Here obſerve, that after dividing by 10 the Remainder is 8, which are 8 *Tenths* of a Pound, that is 16 Shillings; and after dividing by 20 the Remainder is alſo 8, but then it is 8 *Twentieths* of a Pound, that is 8 Shillings. The Learner muſt always conſider well the Value of a Remainder, or he will fall into frequent Miſtakes.

The ſame Queſtion, and (others of the like Nature) may be anſwered as eaſily by multiplying the given Number by the Number of Shillings in the Price, and then dividing by 20, as you ſee in this Inſtance:

288
3
―――
86|4
―――
Anſwer 43*l.* 4*s.*

At 2s. 4d. ½ *a Pound, what will* 141 *Pounds coſt?*

141

$\frac{1}{10}$ for 2*s*.	14—2—0
$\frac{1}{6}$ of 2*s*. for 4*d*.	2—7—0
$\frac{1}{8}$ of 4*d*. for 2*q*.	0—5—10$\frac{1}{2}$
Anſwer	16—14—10$\frac{1}{2}$

In this Example, after dividing by 10 there remains 1, which being 1 Tenth of a Pound, that is, 2 Shillings, I accordingly ſet down 2, and the Line ſtands as 14 Pounds 2 Shillings. Then dividing 14 by 6 I find the Remainder of 2, which being Pounds I conſider as 40 Shillings, and joining them to the firſt remaining 2, I ſay the 6's in 42 are 7, and therefore put down 7 in the Place of Shillings. This Line then is 2*l*. 7*s*. or 47 Shillings, one Eighth of which is 5*s*. 10*d*. $\frac{1}{2}$: And theſe three Sums being added together give the Anſwer to the Queſtion, as above expreſſed.

Once more: *What will* 375 *Bundles of Paper coſt, at* 17s. 6d. *each Bundle ?*

	375
½ for 10*s.*	187—10—0
½ of 10*s.* for 5*s.*	93—15—0
½ of 5*s.* for 2*s.* 6*d.*	46—17—6
Anſwer	328— 2—6

The Manner of working in this Example will be eaſily underſtood by a due Attention to the laſt, and therefore needs no particular Explanation. But it may not be amiſs to obſerve, that the above Queſtion, and others of the ſame Sort, conſiſting of Shillings and Pence, may be anſwered another Way, which in ſome Caſes may be more convenient and eaſy, *viz.* by multiplying the given Number by the Number of Shillings, and then working for the Pence as before. For Example:

At 9s. 9d. *a Yard, what will* 141 *Yards coſt* ?

141
Multiplied by 9

1269
½ of 141 for 6*d*. 70—6
½ of that for 3*d*. 35—3

Total 1374---9

Anſw. 68—14—9

In this Caſe it would have been the ſame Thing, after multiplying by 9 for the Shillings, to have divided the Product by 12 for the Nine-pence, becauſe 9 Pence are a 12th Part of 9 Shillings. And ſo there is ſcarce any Queſtion but may be anſwered by different Operations, as the ingenious Scholar will find by his own Obſervation.

When the Price is any Thing between one Pound and two, let the given Number ſtand for the Pound, and take Parts for the Shillings and Pence. For Example:

At 1l. 2s. 6d. *per Quarter, what will* 286 *Quarters of Malt coſt*?

	286
½ for 2*s*. 6*d*.	35---15

Anſwer 321*l*. 15*s*.

If the Price conſiſts of Pounds, Shillings and Pence, multiply the given Number by the Number of Pounds, and for the Shillings and Pence work by the former Rules. For Example:

At 2l. 17s. 6d. *the Hundred, what will* 144 *Hundred coſt?*

	144
For 2*l.* multiply by	2
	288
½ of 144 for 10*s.*	72
½ of that for 5*s.*	36
½ of that for 2*s.* 6*d.*	18
Anſwer	414*l.*

When with the given Quantity (ſuppoſe a Hundred wt.) there are any odd Parts, as ¼, ½, *&c.* ſuch Parts muſt be taken from the given Price in the Courſe of the Operation; as may be underſtood by the following Example:

At 5l. 5s. *the Hundred, what will* 365*C.* ½ *coſt?*---Here, for the Pounds, I mul-

I multiply the given Number by 5, then take ¼ of it for the 5 Shillings, and ½ of the given Price for the ½ Hundred, and the Sum Total gives the Anſwer to the Queſtion, as appears by the Work at length.

For 5*l.* mult. by 365
5

1825
¼ of 365 for 5*s.* 91--- 5
½ of given Price 2---12---6

Anſwer 1918*l.* 17*s.* 6*d.*

N. B. If the odd Parts had been ¾, after taking half of the given Pric I muſt have taken half of that, *viz* 1*l.* 6*s.* 3*d.*——And hence you may judge how to proceed in Caſes of th like Nature.

By the foregoing Rules of *Practic*
al

all Sorts of foreign Coin may be reduced to Pounds Sterling, of which I need only give one Example.

What is the Amount of 716 *Moidores at* 1l. 7s. *each?*—Here the given Numbers ſtand for the Pound, $\frac{1}{4}$ of which muſt be taken for 5*s.* and $\frac{1}{10}$ for 2*s.* and all added together are the Anſwer to the Queſtion.

	716
$\frac{1}{4}$ for 5*s.*	179
$\frac{1}{10}$ for 2*s.*	71—12
Anſwer	966*l.*—12*s.*

As it is impoſſible for me to ſpeak of the various and almoſt innumerable Methods and Rules of Practice, I have choſen ſuch as I thought were moſt intelligible and uſeful; to whicn I ſhall only add two or three very compendious Ways of working in ſome Caſes, which deſerve the Learner's Notice.

In any Queſtion when the given Price is an even Number of Shillings, multiply the given Quantity by half that Number, doubling the Units of the firſt Product for Shillings, and the other Figures of the Product will be Pounds. For Example:

What will 276 *Ells of Holland coſt, at* 8*s. an Ell?*—Proceeding according to the Rule, the Work will appear as follows:

Multiply	276
by ½ of 8	4
Anſwer	110*l.* 8*s.*

Here I multiply the given Number of Ells by 4, which is half the Number of Shillings in the given Price, ſaying 4 times 6 is 24; but inſtead of putting down 4, the Units of the

Product, I put down twice as many, *viz.* 8, which are the Shillings in the Price required. Then I proceed as uſual, ſaying 4 times 7 is 28, and 2 that I carry is 30, ſet down 0 and carry 3; 4 times 2 is 8, and 3 is 11, which being put down gives the Number of Pounds required, *viz.* 110. and the Operation is finiſhed.

If the Price be an odd Number of Shillings, work for the even Number as before; and for the odd Shillings take one Twentieth of the given Number, and add to the Product. For Example:

At 17s. *a Yard, what will* 172 *Yards coſt*?—Obſerving your Directions, your Work and Anſwer will be as follows:

	172
Mult. for 16*s.* by	8
Product	137—12
$\frac{1}{20}$ for 1*s.*	8—12
Anſw.	146*l.* 4*s.*

If the Price be 2 Shillings, the Queſtion is anſwered by doubling the Units (*i. e.* the right Hand Figure) of the given Number, which are to be reckoned as Shillings, and the reſt of the Figures will be Pounds. For Example:

596 Geeſe at 2*s.* each.

will coſt 59*l.* 12*s.*

Q. How are Sums in *Practice* to be proved?

A. As they may be wrought different Ways, the one may be a Proof of the other; or they may be proved by the *Rule of Three.*

Of TARE *and* TRETT.

After laying down the Rules of *Practice*, it is uſual for Authors to ſpeak of TARE and TRETT, which are Allowances made by Merchants in ſelling their Commodities.

Tare is an Allowance made to the Buyer for the Weight of the Caſk, Bag, Cheſt, &c. in which the Goods are contained; and this is either regulated by Cuſtom, or by a particular Agreement between the Buyer and Seller.

Trett is an Allowance of 4lb. *per* 104lb. made in ſeveral Sorts of Goods for Waſte, Duſt, &c. after the *Tare* has been deducted.

There is alſo another Allowance of 2lb. for every 3 Cwt. called CLOFF or CLOUGH, which is ſometimes made

to

to Retailers for the Turn of the Scale, after the Deduction of the former Allowances.

N. B. The whole Weight, before any of theſe Allowances are made, is called *Groſs*; but when they are deducted, or ſo many of them as are cuſtomary, the Remainder is called *Neat* or *Nett*.

If the Tare of any Quantity of Goods be ſo much in the Whole, and no other Allowance, the *Neat Weight* is found by ſubtracting the Tare from the *Groſs Weight*; but if the Tare be at ſo much *per* Cheſt, &c. multiply the Pounds Tare by the Number of Cheſts, ſubtracting as before; and if at ſo much *per* Cwt. take ſuch Part or Parts of the Groſs Weight, as the Tare is of a Hundred.

In order to work Queſtions of this Kind by the Rules of Practice, it is neceſſary to remember the *even* Parts of a Cwt. (that is 112lb.) and of a Quarter, which are as follow:

$$\begin{array}{l} \textit{lb.} \end{array}$$

$$\left.\begin{array}{r} 7 \text{ is } \frac{1}{16} \\ 8 - \frac{1}{14} \\ 14 - \frac{1}{8} \\ 16 - \frac{1}{7} \end{array}\right\} \textit{of a } \text{C.}$$

$$\left.\begin{array}{r} 1 \text{ is } \frac{1}{28} \\ 2 - \frac{1}{14} \\ 4 - \frac{1}{7} \\ 7 - \frac{1}{4} \\ 14 - \frac{1}{2} \end{array}\right\} \textit{of a } \text{Qr.}$$

As the *Trett* is always 4lb. per 104, the Method of finding it is to divide the Weight by 26, after the Tare is deducted, becauſe 4 times 26 make 104; and then ſubtracting it, the Remainder is the *Neat Weight* required.

Clough, (which is 2*lb.* for 3 *C.* may be found by dividing the Line from whence it is to be deducted by 168, becauſe 2*lb.* is the 168th Part of

336*lb.* or 3 Hundred Weight.

Having laid down theſe Rules, I ſhall only give three plain Examples of the Manner of ſubtracting *Tare* and finding out the ***Nett Weight*** of any Quantity of Goods; leaving the Deduction of *Trett*, *Clough*, or whatever Allowances may be cuſtomary, to the Scholar's own Ingenuity.

If 15 C. 2qrs. 13lb. *Tare be allowed in* 456 C. 1 qr. 19lb. *of Tobacco, what will be the Neat Weight?*

	C. *qrs.* *lb.*	
From	456—1---19	Groſs
Subtract	15---2---13	Tare.
Rem.	443---3---6	Neat.

Here I ſay, 13 from 19 and there remains 6; 2 from 1 cannot, but 2 from 5 (borrowing 4 *qrs.* that is, 1 *C.*) and there remains 3; then, paying the *C.* that I borrowed, I ſay, 1 and

5 is 6, which take from 6 and 0 remains, 1 from 5 and there remains 4, 0 from 4 and there remains 4; and ſo the Work is finiſhed, the Remainder being the Anſwer to the Queſtion.

Again: *What is the Neat Weight of* 3 *Frails of Raiſins, weighing all together* 10 C. 3qrs. 2lb. *Groſs, the Tare* 20lb. per *Frail*?

C.	*qrs.*	*lb.*		*lb. per Frail.*
10—	3—	2	Groſs,	20
	2—	4	Tare.	3
10—	0—	26	Neat.	60 or 2*qrs.* 4*lb.*

Having firſt found the Tare by multiplying the Pounds allowed on each Frail by the Number of Frails, *viz.* 3, I ſet it down under the Groſs Weight, and ſubtract as before, ſaying, 4 from 2 I cannot, but 4 from 30 (borrowing 28*lb.* or *qr.*) and there remains 26; then, 1 that I borrow'd and 2 is 3, which take from 3, and o remains; laſtly, o from 10 and there remains 10; which being ſet down ſhews the Neat Weight 10*C.* 26*lb.* as above expreſſed.

Once more: *Of* 246C. 3qrs. 12lb. *Groſs. Tare* 14lb. per *Hundred, what will be the Neat Weight?*

	C.	qrs.	lb.	
	246—	3—	12	Groſs.
$\frac{1}{8}$ for 14*lb.*	30—	3—	12	Tare.
	216—	o—	o	Neat.

Here, according to *Practice*, the Pounds Tare *per C.* being an even Part of a *C.* namely, one Eighth, I divide the Groſs Weight by 8, which gives the Tare of the whole Quantity. In doing this I ſay, the 8's in 24 are 3, therefore ſet down 3; the 8's in 6 none, and therefore ſetting down a Cypher, I conſider how many Quarters the 6 *C.* contain, which are 24, and theſe carried to the 3 make 27. Then I ſay, the 8's in 27 are 3, which I ſet down, and the Number of Pounds contained in the Overplus 3*qrs*. I add to the 12 Pounds, making 96 in all; and there being juſt 12 times 8 in 96, I ſet down 12 and find the Tare to be 30*C*. 3*qrs*. 12*lb*. as above expreſſed. This being ſubtracted from the *Groſs*, the Remainder is 216 *C.* the *Neat Weight* required.

I hope theſe few Examples will be ſufficient to give the Scholar a Notion of the Method of *ſubtracting* and *dividing* in Queſtions of this Nature, which are the two chief Operations in finding the *Tare*, *Trett*, or other Allowance in any Quantity of Goods, and conſequently the *Neat Weight* after ſuch Deductions.

Here, in Compliance with Cuſtom, I might add ſeveral other Rules, as *Barter*,

Barter, *Fellowſhip*, *Profit and Loſs*, *Alligation*, &c. But as Queſtions in all theſe Rules are anſwered either by the *Rule of Three*, or the *Rules of Practice*, I think they would be an uſeleſs if not a burthenſome Addition to our little Treatiſe; for he who thoroughly underſtands the *Rule of Three*, and thoſe of *Practice*, will ſcarce find any Difficulty in ſolving whatever Queſtions are neceſſary in Trade or the common Affairs of Life. However, I ſhall ſay ſomething of another Rule, which, though of no great Uſe, may be an agreeable Amuſement to the young Scholar, and ſerve to exerciſe his Ingenuity; I mean the Rule of *Poſition*.

CHAP. IX.

Of POSITION *or the Rule of* FALSE.

Q. WHY is this Rule called *Falſe*?

A. Not from its being in itſelf really erroneous, but becauſe we make uſe of *falſe* ſuppoſed Numbers to find out the true Numbers ſought.

Q. Why is it called *Poſition*?

A. It is the ſame as *Suppoſition*, from the *ſuppoſed* Numbers.

Q. Is it not divided into two Parts?

A. Yes; it is uſually divided into two Parts, *Single* and *Double*.

Q. What is the Nature of *Single* Poſition?

A. In the *Single* Rule we uſe but one ſuppoſed Number to find out the Truth, as in the following Example:

Three Perſons, A, B, C, *bought a Quantity of Lead, to the Value of* 140*l.*

of which B *paid as much again as* A, *and* C *as much again as* B. *How much did each pay?*

Suppoſe *A* paid 10*l.* then *B* muſt pay 20*l.* and *C* 40*l.* the Total of which is 70*l.* but ſhould be 140*l.* —Now as the Total ariſing from the Error is to the true Total, ſo is the ſuppoſed Part of the true Part. Therefore, in this Example, as 70*l.* is to 140*l.* ſo is 10*l.* to the Sum that *A* really paid which of conſequence muſt be 20*l.* for 70 is the Half of 140, and 10 is the Half of 20. But the Proportion of the ſeveral Numbers to each other being not always diſcernible at Sight, we muſt work in Queſtions of this Kind by the *Rule of Three*, in order to obtain the Anſwer. In the preſent Caſe therefore I ſay, *If* 70l. *ſhould be* 140l. *what ſhould* 10l. *be?* Which I work as follows:

If 70—140—10
10

70)1400

20

Here multiplying the ſecond and third Numbers together, and dividing the Product by the firſt, according to the *Rule of Three*, the Quotient is 20, which is the Number of Pounds that *A* has paid, as appears by ſetting down the ſeveral Shares as the Queſtion requires.

l.
20 for *A*.
40 for *B*.
80 for *C*.

Total 140

Take one Example more: *A Man overtaking a Maid who was driving a Flock of Geeſe, ſaid to her, Good-morrow, Sweetheart, whither are you going with your* 99 *Geeſe?* *Sir,* ſaid ſhe, *you miſtake the Number; for if I had as many more, and half as many more, and one fourth Part as many, then I ſhould have but* 99. The Queſtion is, how many Geeſe ſhe had?

Let us ſuppoſe ſhe had 40, then as many, and half as many, and one Fourth as many, will make 110, which ſhould have been 99. Therefore ſay, *If* 110 *ſhould be* 99, *what ſhould* 40 *be?* Or, *If* 110 *come from* 40, *what will* 99 *come from?* For it matters not whether 40 or 99 is the middle Number in Stating, ſince they are to be multiplied together. See the Work at length:

If 110—40—99
40

110)3960(36
330

660
660

0

Here the Quotient is 36, which will appear to be the Number of the Woman's Geeſe, by ſetting down the ſeveral Numbers according to her Calculation.

	36 Geeſe.
As many more	36
½ as many	18
¼ as many	9
Total	99

Q. What is the Nature of *Double* Poſition?

A. When there is no Partition in the Numbers to make a Proportion, the *Double* Rule is uſed, wherein we make *two* Suppoſitions; and if with either of them we find the Numbers that ſolve the Queſtion, there is no more to be done; but if, as it commonly happens, we err in both, ſet down the Suppoſitions, and over-againſt them their reſpective Errors, marked thus + if too much, and thus — if too little. Then multiplying them croſs-ways, that is, the firſt Suppoſition by the ſecond Error, and the ſecond Suppoſition by the firſt Error; if both the Errors are alike, *i. e.* both too little, or both too much, ſubtract the leſſer Product from the greater, and divide the Remainder by the Difference of the Errors; but if the Errors are unlike, *i. e.* one too little, and the other too much, the Sum of theſe Products muſt be divided by the Sum of the Errors: In either Caſe the Quotient will give a true Anſwer to the Queſtion, as will appear by the following Examples.

Three Merchants, A, B, C, *built a* Ship, *which coſt* 1600*l.* *of which* A *paid a Sum not known*, B *paid as much again as* A *within* 50l. *and* C *as much as* A *and* B *within* 100l.—The Queſtion is, What did each Man pay?

Now ſuppoſe *A* paid	200*l.*
Then *B* paid - - - -	350
And *C* paid - - - - -	450
Total	1000

But it ſhould have been 1600, therefore the Error is 600 too few.

Suppoſing again that *A* paid	250*l.*
Then *B* paid - - - - -	450
And *C* paid - - - - - -	600
Total	1300

Which Sum is ſtill too little by 300. Therefore ſetting down the Suppoſitions and their Errors in the Manner before directed, they will ſtand thus:

Suppoſitions.	*Errors.*
200	600
250	300

Then multiplying the firſt Suppoſition by the ſecond Error, the Product is 60000; and the ſecond Suppoſition multiplied by the firſt Error produces 150000; the leſſer of which Products being ſubtracted from the greater, the Remainder is 90000; and this being divided by 300, the Difference of the Errors, quotes 300, the Sum that *A* paid, *A* paid, as will appear by adding to it the Shares of *B* and *C* according to the Queſtion.

A paid	- -	300
B paid	- -	550
C paid	- -	750
Total		1600

In this Example the Errors were both alike, *i. e.* both too little; in the following they are unlike, *i. e.* one too little, and the other too much.

When firſt the Marriage-Knot was ty'd
Betwixt my Wife *and me,*
My Age did her's as far exceed
As three times three does three :
But *after ten and half ten Years*
We Man *and* Wife *had been,*
Her Age came up as near to mine,
As eight is to ſixteen.

Now pray,

What were our Ages on the Wedding-Day?

Firſt, ſuppoſe the Wife's Age to be 21 Years, the Huſband's muſt be 63; then adding 15 to each, her Age becomes 36, and his 78. This Error therefore is 3 Years too few.

Again: Suppoſe ſhe was 9 Years old, he muſt have been 27; and by adding 15 to each, her Age is made 24, and his 42. Here the Error is 3 too many.

Therefore ſetting down the Suppoſitions and their Errors, with their reſpective Marks, as above directed, they appear thus:

Suppoſitions.		*Errors.*
21	—	0
9	+	3

Then multiplying croſs-ways, 3 times 21 is 63, and 3 times 9 is 27, theſe Products added together (becauſe the Errors are unlike) make 90 for a Dividend; which divided by 6, the Sum of the Errors, quotes 15, the Wife's true Age when married, as is thus proved:

She being 15, he muſt be 45
15 added to each 15
Make her 30, half of his 60

I forbear giving more Examples of this Nature, as the Scholar may eaſily frame them himſelf: but I ſhall add a Queſtion or two in *Progreſſion*, which will not be diſpleaſing to thoſe who take Delight in the Art of Numbers.

CHAP. X.

PROGRESSION, *briefly explained, with ſome Examples.*

Q. IS not this Rule divided into two Parts?

A. Yes: Progreſſion is either *Arithmetical* or *Geometrical.*

Q. What is *Arithmetical Progreſſion*?

A. It is the regular Increaſe or Decreaſe of any Series of Numbers by the continual Addition or Subtraction of ſome equal Number. So 1, 3, 5, 7, 9, and 35, 28, 21, 14, 7, are two Ranks of Numbers in *Arithmetical Progreſſion*; the firſt increaſing by the continual Addition of Two, and the ſecond decreaſing by the continual Subtraction of Seven.

Q. What is *Geometrical Progreſſion*?

A. It is the Increaſe or Decreaſe of any Series of Numbers by an equal *Ratio*, that is, by the continual Multiplication or Diviſion of ſome equal Number. Thus 2, 4, 8, 16, increaſe in Geometrical Progreſſion by a *double Ratio*, or a continual Multiplication by 2; and 135, 45, 15, 5, decreaſe in the ſame Manner in a *triple Ratio*, or a continual Diviſion by 3.

Q. What is chiefly to be regarded in *Progreſſion*?

A. The five following Things, *viz.* 1. The firſt Term; 2. The laſt Term; 3. The Number of Terms; 4. The *Ratio*, or equal Difference; 5. The Sum of all the Terms.——By knowing ſome of theſe Things the reſt are found according to various Rules laid down by Arithmeticians, too numerous to be here tranſcribed and exemplified; and therefore we refer the Curious to larger Treatiſes for Satisfaction in theſe Particulars, which would ſwell our little Work beyond its intended Limits. However, we ſhall give the few Examples promiſed, and ſo conclude.

A Grazier having brought 23 *fine Oxen to Market, a Butcher offers him* 16l. *a piece, and take them all. The Grazier refuſes the Money; but, ſays he, If you'll give me what the laſt Ox will come to, reckoning the firſt at a Farthing, the ſecond at a Halfpenny, the third at a Penny, and ſo doubling the Price through the whole Number, you ſhall have them: To which the Butcher readily aſſents, thinking he had made an excellent Bargain.* The Queſtion is, What did the Butcher pay according to this Agreement for the 23 Oxen?

Here obſerve, that the firſt Farthing is to increaſe *geometrically* in a double *Ratio*, that is by 2, till the Number of Terms amount to 23. But as it would be a troubleſome and tedious Operation to multiply the ſeveral

Terms by 2 from the firſt to the laſt, in order to find out the Price required, ſet down a few of the leading Terms, and place the Exponents over them in the following Manner:

0 . 1 . 2 . 3 . 4 . 5
1 . 2 . 4 . 8 . 16 . 32

The upper Row of Figures, called *Exponents*, ſerve to ſhew the Diſtance of the Terms from Unity, or from the firſt Term of the Series: But obſerve, that as the Exponents are leſs by one than the Terms, the Term we are in ſearch of, *viz.* 23, will anſwer to the Exponent 22.--Now the Rule for finding any diſtant Term, after a few of the leading ones are ſet down, is this: Multiply the laſt found Term by itſelf, and it will produce a Term double thereto; which again multiplied by itſelf will produce another double to the laſt: And thus proceed till either you produce the Term ſought, or one a little ſhort of it; which may be compleated by multiplying it again by that Term which ſtands under ſuch Exponent as will make up the Number.

In the preſent Caſe therefore you are to multiply the 32 by itſelf, which being the 5th Term (according to the Exponent) produces the 10th, namely, 1024; and this again multiplied by itſelf gives 1048576, the 20th Term from the firſt, but taking the firſt into the Number it is the 21ſt Term: Therefore as two Terms are wanting to make up the Number required by the Queſtion, *viz.* 23, the laſt Product is to be multiplied by the Term under the Exponent 2, which is 4, and this gives 4194304, the 22d Term if reckoned by the Exponents, but in reality the 23d, and conſequently the Price of the Oxen in Farthings, which makes 4369*l.* 1*s.* 4*d.*—See the Work at length:

```
     32
     32
     --
     64
    96
   ----
   1024
   1024
   ----
   4096
  2048
10240
-------
1048576
      4
-------
4194304
```

Hence you may learn how neceſſary it is to have a competent Knowledge of Arithmetic, to prevent being impoſed upon by cunning and deſigning Perſons; for had the Butcher been but a little acquainted with the Nature of Numbers, he had not been overreached by the Grazier, and paid ſuch an extravagant Price for his Oxen.

I ſhall propoſe another Queſtion of the ſame Kind, and give the Anſwer to it, leaving the Scholar to try his Skill in the Operation.

Suppoſe a Nobleman purchaſes a fine Horſe, having 28 *Nails in his Shoes, at the Price of the laſt Nail, valuing the firſt at a Farthing, the ſecond at two, the third at four, and ſo doubling the Number of Farthings for each Nail, till the* 28 *are reckoned.* The Queſtion is, What was the Value of the laſt Nail, *i. e.* the Price of the Horſe?---*Anſwer*, 134217728 Farthings, which is 139810*l.* 2*s.* 8*d.*

Take one Queſtion more of a different Nature, *viz. How many Changes can be rung upon* 10 *Bells*?

The Method of anſwering this is, to ſet down the whole Number of Terms given, (as 10 in this Example, and they will form a Series of Numbers in *Arithmetical Progreſſion*: Then multiply the firſt by the ſecond, that Product by the third, and that Product by the fourth, and ſo on, till you have gone through all the Terms, and the laſt Product is the Anſwer ſought: And by this means you not only find how many Changes can be rung upon 10 Bells, but upon 2, 3, 4, 5, 6, 7, 8, and 9; as appears by the Work:

Bells.	*Changes.*
1	——
2	2
3	6
4	24
5	120
6	720
7	5040
8	40320
9	362880
10	3628800

Every one knows that 1 Bell admits of no Variation, but the Changes the other Numbers are capable of are found by multiplying after this Manner: Twice 1 is 2, 3 times 2 is 6, 4 times 6 is 24, 5 times 24 is 120, *&c.* To make this plain by one Inſtance, let *a b c* be 3 Bells, and you will find they may be changed 6 Ways with reſpect to their Places, and no more, namely, *abc*, *acb*, *bac*, *bca*, *cab*, *cba*.

In like Manner may the Changes of any other Number of Things be found, as of 12 Bells, 24 Letters, *&c.* And this is called *Permutation*.

FINIS.

BRITISH MUSEUM
JUL